XLII. *Obſervations for determining the Length of a Degree of Latitude in the Provinces of* Maryland *and* Pennſylvania, *in* North America, *by Meſſieurs* Charles Maſon *and* Jeremiah Dixon.

Read Nov. 24, 1768.

IN this work, the firſt thing to be conſidered was, how to continue a right line: and this was done by ſetting up marks with the aſſiſtance of an equal altitude or tranſit inſtrument (for it was contrived ſo as to ſerve either purpoſe at pleaſure), made by Mr. John Bird, of the ſame conſtruction with that deſcribed by M. Le Monnier, in the preface to the ſingle volume of the French Hiſtoire Celeſte.

The cylindrical ends of the croſs axis of the teleſcope were laid in two angles of the ſupporters, which roſe perpendicularly from a horizontal bar, that was faſtened firmly to the upper part of the vertical axis. The axis of the teleſcope was ſet truly horizontal, by a ſpirit level hung on its cylindrical ends.

The braſs frame, which receives the vertical axis, was ſcrewed to a poſt fixed in the ground, in the direction of the line which was to be continued.

When the vertical wire in the teleſcope was brought to biſect any mark, it was kept in that direction, by confining firmly, between two puſhing ſcrews, a horizontal arm that projected from a collar that ſurrounded the vertical axis; and, to prove that a ſmall ſhock would not alter its poſition, a ſmall preſſure was applied againſt one of the ſupporters, which being removed, it was carefully noted, whether the wire returned again to biſſect the mark.

At every ſtation (or mark) the teleſcope was turned two or three times after the mark was fixed in the line, to prove that the ſaid mark was truly ſet.—In general, the diſtances between the marks did not exceed a mile, nor were they leſs than half a one.

The teleſcope magnified about 25 times. Three or four marks were always left ſtanding, and on a little riſing ground they would all be ſeen in a right line, the vertical wire in the teleſcope biſſecting their centers without ſenſible error.

The marks made uſe of in continuing the lines were concentrical circles of black and white, painted upon both ſides of a board 14 inches ſquare. This board moved in mortices made in two poſts, which were drove into the ground; and, when the center of the ſaid mark was brought, by means of ſignals, into the line, it was faſtened by wedges to the poſts.

By means of a plummet, a peg was driven into the ground, and a notch cut in it, under the center of the ſaid mark, in order to ſecure the line.

In the evening, when we left off, a mark was placed before, and two or three left behind us; and in the morning the inſtrument was again ſet up in the ſame place, to prove that the marks were not moved.

The tremour of the air (cauſed by the ſun's rays) was often very great; and, to avoid any error that might ariſe from the fluttering of the marks, we intermitted our operations ſometimes for five or ſix hours in a day, and were often obliged to make uſe of the morning or evening twilight.

In the continuation of the line, a perſon was left at the mark, behind the inſtrument, till another mark was ſet forward, to prove with a plummet that its center was not moved.

The viſto cut through the woods, in this work, was about eight or nine yards wide, and, in general, ſeen about two miles, beautifully terminating to the eye in a point.

The zenith diſtances of the ſtars, for determining the celeſtial arc, anſwering to the interval of the parallels of the northernmoſt and ſouthernmoſt points of the lines, were made with an excellent ſector of ſix foot radius, conſtructed by Mr. John Bird.

In the courſe of the work, for dividing the provinces of Maryland and Pennſylvania, the following lines were traced out, that offered themſelves for determining the length of a degree of latitude.

In the following fig. let N repreſent the northernmoſt point, and A the moſt ſouthern of the ſaid lines. Beginning at N, a meridian was traced from N to P. $= \overset{\text{mil.}}{14}\ \overset{\text{cha.}}{64}\ \overset{\text{lin.}}{8}$. In this line there were ſome hills, which were meaſured horizontally with a level, but the plains were meaſured with a chain.

P C $= \overset{\text{m.}}{2}\ \overset{\text{ch.}}{79}\ \overset{\text{lin.}}{27}$; C being in the parallel of latitude with P, which was determined by the ſector.

D C a meridian $= \overset{\text{m.}}{5}\ \overset{\text{ch.}}{2}\ \overset{\text{lin.}}{43}$, in which are three or four ſmall aſcents and deſcents.

The

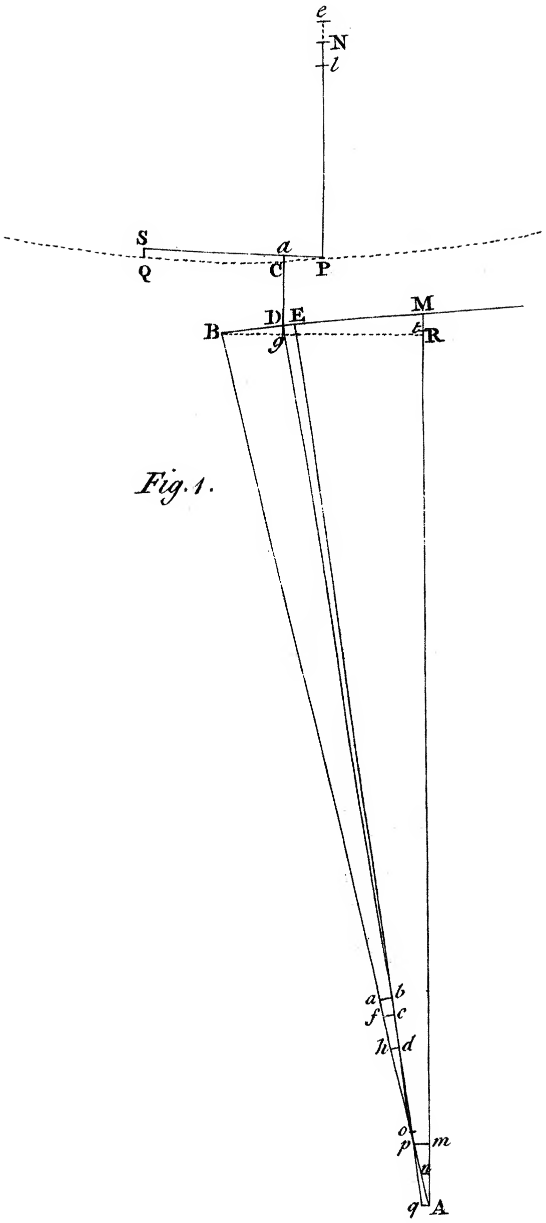


Fig. 1.

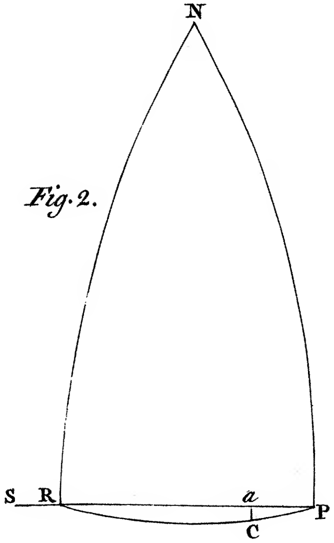


Fig. 2.

The points B D E and M are in a right line. $BD = \overset{\text{cha.}}{22}\ \overset{\text{lin.}}{51}$, and the angle $CDM = 86° \ 32\tfrac{1}{7}'$ nearly. Hence,

B is ſouth of $D = \overset{\text{ch.}}{1}\ \overset{\text{lin.}}{36} = Dg$.

The line $AB = \overset{\text{m.}}{81}\ \overset{\text{ch.}}{78}\ \overset{\text{lin.}}{31}$, in which is one gentle riſing hill, about half a mile over; all the reſt of the line is an entire level or plain.

Theſe meaſurements, expreſſed in Engliſh ſtatute miles and parts of the ſame, were made with a chain, eſtabliſhed from a braſs ſtatute yard, which was proved and corrected, in the courſe of the work, by another ſtatute chain (kept only for that purpoſe) made from the ſaid braſs yard. They were only deſigned for dividing the provinces of Maryland and Penſylvania: the ſame lines were re-meaſured afterwards with wooden rectangular levels, for the purpoſe of determining the length of a degree of latitude, as will appear in the ſequel of this work.

The point C was placed in the parallel of latitude of P, thus. Let N (ſee TAB. XIII. fig. 2.) repreſent the north pole of the terreſtrial globe; P and R two places lying in the ſame parallel of latitude R C P; P R an arch of a great circle$=10'$ joining the ſaid points; and P N, R N two meridians. P N or the complement of the latitude of P being $= 50° \ 16' \ 42''$, the angle N P R or the azimuth of the great circle P R was found by calculation to be $89° \ 55' \ 51''$. The going of the clock being found by equal altitudes of ſtars, the times were computed when the ſame or other ſtars would paſs the azimuth of the line P R; and, at the time computed for any ſtar, the interſection of the croſs wires of the tranſit inſtrument being brought to cover the ſtar, the teleſcope was turned down to the horizon, and a land-mark was fixed up at the diſtance of about half a mile, anſwering to the interſection of the wires. In like manner, by other ſtars, ſeveral other marks were fixed up, and the mean of all was taken. In this direction the line P R was continued; and though it was at firſt intended to extend it only to R, to the diſtance of 10′ of a great circle, it was in fact prolonged ſomewhat further, to S, P S being = 12,312 miles, or 10′ 45″ of a great circle. Now P C being = 2,991 miles, or 2′ 37″ of a great circle, the angle N P C is = 89° 58′ 55″; from whence N P S = 89° 55′ 51″ being ſubtracted, there remains the angle S P C or *a* P C = 3′ 4″, whence *a* C, or the diſtance of the parallel P C R at C, ſouth of *a*, ſhould be 14,1 feet. But it having been made a rule, in dividing the provinces of Pennſylvania and Maryland, to trace out the parallels of latitude by the obſervations taken with the aſtronomical ſector only, the ſector was put up at P and S ſucceſſively (ſee fig. 1.) and the zenith diſtances of the ſtars Capella α Lyræ, and others, were obſerved at both places; whence the point S was concluded to be 43 yards or 129 feet = S Q more northerly than P; and thence it was found by calculation, that the parallel of latitude P Q at the point C ſhould be $45\frac{1}{2}$ feet, = *a* C diſtant from the great circle P S, and to the ſouth of the ſame; and the point C was placed accordingly, bp laying off $45\frac{1}{2}$ feet = *a* C, at right angles, to the line P *a* from the point *a* towards the ſouth.

a C found by the ſector, being $45\frac{1}{2}$ feet, and found by the azimuth of the line P S being 14,1 feet only, it follows, that had the poſition of the point C been determined by the latter method, inſtead of the former, it would have been placed 31,4 feet more to the northward than it was found by the ſector; and, in conſequence, the length of the degree of latitude would have come out 21 feet longer. But the difference is ſo ſmall, that it only ſerves to confirm the exactneſs of the work, and renders it unneceſſary to enter into any conſideration, which of the two methods ought to be preferred.

The meridians NP, CD, and AM, were found by celeſtial obſervations. The method of proceeding was as follows:

To find the meridian AM, and the angle that the line AB makes with the ſaid meridian.

The equal altitude inſtrument being ſet up at the point A, with its vertical axis over the ſaid point, equal altitudes of ſtars were obſerved for finding the motion of the clock. The time was next computed when ſome northern ſtars would paſs the meridian by the clock, at which inſtant (ſhewn by the clock) the vertical wire in the teleſcope was brought to biſect the ſtar; and, the vertical axis of the inſtrument remaining fixed, the teleſcope was turned down in the ſame azimuth to the horizon, and a candle placed oppoſite to the vertical wire, as a point in the meridian.

And the time of ſtars paſſing an azimuth in the direction of the line AB, for determining the angle BAM, was found by bringing the vertical wire in the teleſcope to biſect a candle placed (about $1\frac{1}{4}$ mile from A) in the line AB; the teleſcope was then elevated to the ſtar, and the time when it paſſed the ſaid vertical wire taken.

The obſervations for determining the meridian AM, and the angle that the line AB makes with the ſaid meridian, were as follows:

1766. October.	Time per clock.		Sum.	Half Sum.	
	h ′ ″	h ′ ″	h ′ ″	h ′ ″	
♄ . . . 11	19 51 27	21 17 10+	41 11 23+	20 35 41½+	equal altitudes α Cygni.
	52 49—	18 35+	11 24	35 42	
	54 13	19 56	11 23	35 41½	
			Mean —	20 35 42	ſtar paſſed the meridian per clock.
				20 33 30	ſtar's apparent right aſcenſion.
				2 12	clock too faſt for ſidereal time.
☉ . . . 12	20 0 16	21 10 22::	. . . .	. . . .	equal altitudes α Cygni.
	1 42	11 47—	41 13 29—	20 36 44½	
	3 15	13 14	13 30	36 45	
			Mean —	=20 36 45—	ſtar paſſed per clock.
				20 33 30	ſtar's apparent right aſcenſion.
				3 15—	= clock too faſt for ſidereal time.

22 20 13 α Urſa major } paſſed an azimuth in the direction of the line AB.
23 26 β Urſa major }

1766. October.	Time per clock. h ′ ″	h ′ ″	Sum. h ′ ″	Half Sum. h ′ ″	
☽ . . . 13	19 50 41	21 22 12+	41 15 33—	20 37 46½	equal altitudes α Cygni.
	51 58	23 35½	15 33½	37 47—	
	53 20½	24 52	15 33	37 46½	
			Mean	= 20 37 46½	ſtar paſſed per clock.
				20 33 30	ſtar's apparent right aſcenſion.
				4 16½	Clock too faſt. Hence the clock gained of ſidereal time the laſt 24^h. . . 62″.

22 21 16 α Urſa major } paſſed an azimuth in the direction of the line A B.
22 24 21 β Urſa major }

	h ′ ″
The apparent right aſcenſion of α Urſa major	22 49 8
The clock will be too faſt, when this ſtar will be on the meridian	4 22—
α Urſa will be on the meridian per clock	22 53 30—
In like manner I find that γ Urſa major will be on the meridian per clock . . .	23 45 55½
And the pole ✻ at	0 50 27

At the inſtant when the clock ſhewed $22^h\ 53'\ 30''$, the vertical wire was brought to biſſect the ſtar α Urſa major; and then the vertical axis was made faſt (the level ſhewing the horizontal poſition of the axis of the teleſcope and the line of collimation being juſt), the teleſcope was then brought down to the horizon, and by means of a candle ſeen through a ſmall hole in a board, a mark, at the diſtance of $\overset{\text{cha.}}{21}\ \overset{\text{lin.}}{42}$, was placed in a line with the ſaid vertical wire.

In like manner, when γ Urſæ majoris, and the Pole ſtar paſſed the meridian, as known by the clock, the vertical wire was brought, at thoſe inſtants of time, to the ſaid ſtars, and the teleſcope turned down to the horizon, in the ſame manner above mentioned, and the wire biſſected the candle, when held to the 1ſt mark, as near as could be judged.

1766. October.	Time per clock.		Sum.	Half Sum.	
	h ′ ″	h ′ ″	h ′ ″	h ′ ″	
♂ . . . 14	20 2 21	21 12 15	41 17 35	20 38 47½	equal altitudes of α Cygni.
	3 48	13 47	17 35	38 47½	
	5 20+	15 14	17 35+	38 47½	
				20 38 47½	
				33 30	= ſtars A. R.
				5 17½	Clock faſt.
	23 37 44½	0 20 49	0 3 36	0 1 48	equal altitudes of α Andromeda.
	40 6½	23 32	3 38½	1 49+	
	42 47	25 54	3 38	1 49+	
				0 1 49—	
				23 56 22	
				5 27—	clock too faſt.
				0 51 30	the Pole * will be on the meridian.
				2 57 10½	β Urſa minor on ditto.

2 3 40 β Urſa minor paſſed the line A B.
Cloudy when the Pole ſtar paſſed the meridian.

At 2 57 10½ β Urſa minor was biſſected by the vertical wire for finding a meridian; and the teleſcope then turned down to the horizon, and by bringing a candle, at the diſtance of a mile, to be biſſected by the vertical wire, we there placed a mark. After theſe obſervations, the clock was wound up, in doing which it was ſtopped about 23″.

1766. October.	Time per clock. h ′ ″	h ′ ″	Sum. h ′ ″	Half Sum. h ′ ″	
☿ . . . 15	19 52 14+	21 23 47	41 18 42½	20 39 21+	equal altitudes α Cygni.
	53 22½	25 10	18 42½	39 21+	
	54 55½	20 28	18 42+	39 21	
				20 39 21+	
				33 30	
				5 51+	clock too faſt.

22 31 56 δ Urſa minor paſſed the line A B.
2 4 18 β Urſa minor paſſed ditto.

♃ . . . 16	20 0 17	21 17 24	41 20 32	20 40 16	equal altitudes α Cygni.
	1 41	18 51½	20 32½	40 16+	
	3 8	20 16	20 33	40 16½	
				20 40 16+	
				20 33 30	
				6 46+	clock too faſt.

22 23 46 α Urſa major paſſed the line A B.
22 26 45½ β Urſa major paſſed ditto.
22 32 45 δ Urſa minor paſſed ditto.

1766. October.	Time per clock. h ′ ″	h ′ ″	Sum. h ′ ″	Half Sum. h ′ ″	
♃ . . . 16	23 41 16	0 19 46	0 6 37	0 3 18½	equal altitude of α Andromeda.
	43 51½	22 46	6 37½	3 19—	
	46 51	25 22	6 38	3 19	
			Mean =	0 3 19	✱ paſſed the meridian per clock.
				23 56 22	✱'s apparent A R.
				6 57—	the clock too faſt. Hence by the going of the clock, in the interval between α Cygni and α Andromeda paſſing the meridian.

	h ′ ″	
At	0 53 0	the Pole ✱ will be on the meridian.
	2 58 41	β Urſa minor will be on ditto.
At	0 53 0	by the clock, the vertical wire was brought to the Pole ✱ as uſual; and, by means of a candle at the diſtance of a mile, a mark was placed, which fell, as near as could be judged, on the mark placed the 14th inſtant.
At	2 5 10	β Urſa minor paſſed the line A B.
	2 58 41	the wire was brought to β Urſa minor for finding a meridian as before; and, by means of a candle at the diſtance of a mile, a mark was placed, which fell 3 inches eaſt of that placed the 14th.

N. B. In this laſt obſervation the axis of the teleſcope was turned end for end; that is, the teleſcope itſelf was turned upſide down. This proved the ends of the cylinder to be good.

♀ . . . 17 In the evening, by means of a candle placed behind a board, with a ſmall hole in it over the mark placed the 13th inſtant, the line was extended to the marks at a mile diſtance, and there a mark placed, which fell $\frac{1}{4}$ of an inch eaſt of the mark placed the 14th inſtant.

From the whole, there are ſix obſervations, all within the ſpace of about 3 inches, at the diſtance of a mile: The mean was taken as a point in the meridian, north of the point A.

At this meridian point *m.* we laid off the line *mp.* at right angles to the meridian A, *m*, M, and, by a candle being placed at *o*, in the right line A B (about $1\frac{1}{4}$ miles from A), another candle was advanced along the line *mp*, till the vertical wire in the teleſcope biſſected both candles: Under the candle, at the interſection of *mp*, with A B, viz. at *p*, a mark was placed in the ground.

The ground between *m*, and *p*, being made ſmooth (it was level as a floor by nature) the diſtance *mp*, was meaſured twice, and found to be 5 chains, 14 feet, and $\frac{3}{10}$ of an inch.

With this ſame chain the diſtance *m* A was meaſured = 80 chains exactly.

For the Angle B A M, by celeſtial meaſure.

	h ′ ″		° ′ ″
The AR of the meridian when α Urſa major paſſed an azimuth in the direction of the line A B.	22 16 53, 8	by obſervations made October 12.	
	22 16 55, 1	D° on the . . . . 13.	
	22 16 54, 7	D° on the . . . . 16.	
Mean . . . . =	22 16 54 5	=	334 13 38
AR of α Urſa major			162 16 54
Angle at the Pole = ...			171 56 44

	h ′ ″		
AR's of the meridian when β Urſa minor paſſed the direction of the line AB.	1 58 7, 8	by obſervations made October	14.
	1 58 14, 2	D° on the	15.
	1 58 7, 1	D° on the	16.

		° ′ ″
Mean of the firſt and laſt = ...	1 58 7, 5 = ...	29 31 52
AR of β Urſa minor		222 53 30
Angle of the Pole		166 38 22

Now having the diſtance of the ✳s from the Pole, the angles at the Pole, and the latitude of the point A, per ſpherics, we find the ſtar's azimuth from the north = the angle B A M.

And for the ſaid Angle by terreſtrial meaſurement. As A M : Rad :: pm : Tangent Angle pAm. = the Angle B A M. =

	° ′ ″	
Tangent Angle pAm. = the Angle B A M. = ...	3 43 40	
Angle per α Urſa major	3 43 25	
D° per β Urſa minor	3 43 25	
Mean	3 43 30	= Angle B A M.

In the ſame manner as the point m, in the meridian from A was found, points in the meridian, north of P and N, were alſo found; and having two points given, with them a right line was extended as follows; firſt N P. in a line with N e.——At N the equal altitude inſtrument was ſet up, and the vertical wire in the teleſcope was brought to biſſect the mark at e; and there the vertical axis made faſt. The ſpirit level ſhewing the axis of the teleſcope to be horizontal.

The vertical axis being well ſecured, the teleſcope part is taken off the ſupporters, and turned to point to the ſouthward (carefully taking off, and putting it on the ſupporters ſo as not to move the axis); then on the fartheſt riſing ground that could be ſeen, another mark was placed at l, in a right line with the vertical wire. A mark being left at N, the inſtrument is taken, and ſet up three or four feet ſouth of the mark l, and having brought the vertical wire in the teleſcope in a right line with the marks at l, and N, the vertical axis is then made faſt as before, the teleſcope immediately turned, and a third mark placed to the ſouthward; and ſo the operation was continued.

In the ſame manner the lines P S, C D, A B, and A E were traced out; and, to prove that by this method a right line may be extended, we ſhall here give the reſult of continuing the lines A B and A E. A and D being two points between which a right line was to be drawn.

The point *n* was known to be nearly in the line A D. At A and *n* marks were placed, and 3 or 4 feet north of *n* (which was $\frac{1}{4}$ of a mile from A), the inſtrument was ſet up; then, in the ſame manner as above, the vertical wire in the teleſcope was brought in a line with A and *n*, and the vertical axis made faſt; the teleſcope was then turned to point to the northward, and a third mark placed, &c. &c. In this manner the line A *n* was continued to B.

Having continued the line to B, it fell $\overset{\text{cha.}}{22}\ \overset{\text{lin.}}{51}$, weſt of the point D: we then returned and laid off eaſtward off-ſets from the line A B, at every fifth mile from A, proportional to the diſtance from A, and at the end of every off-ſet placed a poſt, in order to form the line A D.

Off-ſets were meaſured very correct at $\left\{\begin{array}{l} \text{about } 10\frac{1}{2} \text{ miles} \ldots\ldots = ab. \\ \text{the } 10^{\text{th}} \text{ mile} \ldots\ldots\ldots = fc. \\ \text{about the } 9^{\text{th}}\ \text{d}^{\circ} \ldots\ldots = bd. \end{array}\right\}$ and at the 3 points, *b*, *c*, *d*, three marks were placed. At *b* the inſtrument was ſet up, to ſee if the 3 marks were placed in a right line; when it appeared they were not exactly ſo; but, on moving the middle one $\frac{1}{2}$ an inch eaſt, they then made a right line.

In this direction we continued the lines *b*, *c*, *d*, to A, which fell 2 feet 2 inches weſt of the point A, at *q*. The diſtance A *q* being ſo ſmall a quantity (gradually riſing) in ten miles, we thought it would be ſuperfluous to change the direction, and therefore returned to the point *b*, and extended the line northward, proceeding in the ſame manner as before.

Having continued the line to E, it fell 16 feet 9 inches eaſt of the point D.

Hence the off-ſets from this line to the true line A D, are as ſhewn by the Table B. And as we paſſed by the off-ſet poſts made from A B, we meaſured the diſtance of this line A E from the ſaid off-ſet poſts, which were as given in the Table D.

TABLE

TABLE B.

Miles from point A.	Off-ſets in		
	Feet	Inches	
0	2	2	to the eaſtward.
5	1	0	
10	0	2.2	to the weſtward, to give the true line A D.
15	1	4	
20	2	6	
25	3	8	
30	4	10	
35	5	11	
40	7	1	
45	8	3	
50	9	5	
55	10	6	
60	11	8	
65	12	10	
70	14	0	
75	15	2	
80	16	4	
82	16	9	

TABLE D.

Miles from point A.	Off-ſets in		
	Feet	Inches	
0	2	2	eaſtward.
5	0	10	
10	0	0	the diſtance of the off-ſet poſts, weſt from the line A E.
15	0	4	
20	4	8	
25	7	4	
30	8	3	
35	7	6	
40	8	5	
45	9	6	
50	10	11	
55	11	1	
60	10	6	
65	11	7	
70	12	11	
75	15	7	
80	16	7	
82	16	9	

From theſe Tables we have the difference of the reſults of the two lines A B and A E; that is, the off-ſet poſts from the line A B, which form the true line A D: And the off-ſet poſts from the line (A E) traced laſt, that alſo form the ſaid line A D, will be diſtant from each other, at every fifth mile, as follows:

Miles

Miles from point A.	Feet	Inches
0	0	0
5	0	2
10	0	2.2
15	1	0
20	2	2
25	3	8
30	3	5
35	1	7
40	1	4
45	1	3
50	1	6
55	0	7
60	1	2
65	1	3
70	1	1
75	0	5
80	0	3
82	0	0

This is a ſufficient proof that the line A B is the arch of a great circle; but, as a farther confirmation that no error could ariſe, we obſerved at different points in the line the right aſcenſions of the meridian when the ſtar δ Urſæ minoris paſſed an azimuth correſponding to the direction of the line, being in the upper part of its circle.

The method of proceeding was in the ſame manner, as deſcribed before, for finding the angle B A M.

OBSERVATIONS for determining the right Aſcenſions of the Meridian, when δ Urſæ Minoris paſſed the Line A B.

June 25, 1764, we began at the point A, to trace the line A B; and the weather being ſo cloudy prevented our making any one obſervation till July 10, though we attended every night. By this time we had continued the line 20 miles from A.

1764. July.	Time per watch. h ′ ″	h ′ ″			h ′ ″
δ . . . 10	15 49 0	17 5 10	equal altitudes of Antares.	Hence the * paſſed the meridian per watch at	16 33 36
	55 0	17 12 17		A R of Antares - - - - - - - - - - - -	16 15 1
	16 2 8	18 3		Watch too faſt for ſidereal time - - - - - - - - -	0 18 35

	1764. July.	Time per watch. h ' "	h ' "			h ' "
20 Miles from A.	♂ . . . 10	19 29 26	20 15 36	equal alt. α Aquilæ.	Hence * paſſed at . . .	19 57 58
		34 18	21 40		Right aſcenſ. of α Aquilæ	19 39 18
		40 21	26 30			
					Watch too faſt	0 18 40

22 44 45 ♁ Urſæ minoris paſſed an azimuth in the direction of the line A B.
— 18 44 watch too faſt when the * paſſed the line.

22 26 1 = the right aſcenſion of the meridian, when ♁ Urſæ minoris paſſed the line A B.

---

		h ' "	h ' "			h ' "
	27	18 10 45	19 45 4	equal altitude α Lyræ.	Hence * paſſed at . .	18 59 0
		11 54	46 10		Right aſcenſion = . .	18 28 58
		12 56	47 11			
					Watch too faſt . . .	0 30 2
43 Miles from A.		20 33 58	21 30 50	equal alt. α Cygni.	Hence * paſſed at . . .	21 3 58
		35 33	32 23		Right aſcenſion . . .	20 33 25
		37 3	33 58			
					Watch too faſt . . .	0 30 33

22 57 36 ♁ Urſæ minoris paſſed the direction of the line A B.
— 31 1 watch too faſt at this time.

22 26 35 = the right aſcenſion of the meridian when ♁ Urſæ minoris paſſed the line A B.

	1764. July.	Time per clock. h ′ ″	h ′ ″			h ′ ″
	♂ . . . 30	18 32 40	20 2 50	equal alt. of α Lyræ.	Hence * paſſed at . . .	19 18 51
		33 42	. . . . . .		Right aſcenſion	18 28 58
		34 55	1 4 59		Watch too faſt	0 49 58
46 Miles from A.		20 56 12	21 49 13	equal alt. of α Cygni.	Hence paſſed at	21 24 23
		57 56	50 52		*'s right aſcenſion . . .	20 33 25
		59 36	52 26		Watch too faſt	0 50 58
		23 18 25	δ Urſæ minoris paſſed the direction of the line A B.			
		— 51 57	Watch too faſt.			
		22 26 28	=the right aſcenſion of the meridian when δ Urſæ minoris paſſed the line A B.			

	Auguſt.	h ′ ″	h ′ ″			h ′ ″
	. . . 17	18 47 25	19 0 0	equal alt. of α Lyræ.	Hence * paſſed at	19 12 42
		48 24	37 0		its right aſcenſion	18 28 58
		49 33	38 0		Watch too faſt	0 43 44
11 Miles from A.		20 42 45	21 50 48	equal alt. α Cygni.	Hence * paſſed at	21 18 9
		44 8	52 12		its right aſcenſion	20 33 25
		45 30	53 30		Watch too faſt	0 44 44
		23 12 48	δ Urſæ minoris paſſed the direction of the line A B.			
		— 45 39	=watch too faſt.			
		22 27 9	=the right aſcenſion of the meridian, when δ Urſæ minoris paſſed the direction of the line A B.			

1764. August.	Time per watch.						
	h	′	″	h	′	″	
♂ . . . 27	18	54	38	19	58	50	equal altitudes α Lyræ.
		55	40		59	54	
		56	44	20	0	57	
Here we were 80 miles from A.	21	6	32	21	54	46	equal alt, α Cygni.
		8	4		56	25	
		9	44		58	2	

	h	′	″
Hence the * paſſed merid. per watch . . .	19	27	47
Right aſcenſion of α Lyræ	18	28	58
Watch too faſt	0	58	49

	h	′	″
Hence paſſed at	21	22	31
*'s right aſcenſion	20	33	25
Watch too faſt	0	59	6

23	25	55	δ Urſæ minoris paſſed the direction of the line A B.
—	59	21	watch too faſt.
22	26	34	= the right aſcenſion of the meridian when δ Urſæ minoris paſſed the line A B.

Moſt of theſe equal altitudes were obſerved per Mr. Dixon, I judging the time by the watch, which had only the hour and minute hands; therefore the ſeconds muſt not be expected as from a good clock, nor does the problem require it, as the ſtar δ Urſæ minoris changed its azimuth very ſlowly.——The paſſage of the ſtar δ Urſæ minoris over the line A B was in general taken by myſelf.

The A R of meridian when δ Urſæ minoris paſſed A B.	22	26	0½	by obſervations made on the 15th and 16th of October, 1766, at the point A.
	22	25	54½	

Whether the ſmall effects of the aberration and nutation of the ſtar δ Urſæ minoris, at the different times, will add to or diminiſh from the correſpondence of theſe numbers, I have not determined; the above being ſufficient for the purpoſe intended: for if the direction of the line had been changed any quantity of note, it would have cauſed a much greater difference in the right aſcenſions of the meridians, when the ſtar paſſed the line, than any we here find, The following are obſervations for determining the celeſtial arch between the points A and N.—— Thoſe marked with dots and *, thus, . . * . . were made by Mr. Dixon.

1766. October. ☿ 8 We ſet up the ſector at the point A, in the middle of a weſt line, drawn between Cape Hinlopen and Cheſopeak Bay, and made the following obſervations.

N. B. Each revolution of the micrometer = 52″

PLANE OF THE SECTOR EAST.

*'s names.	Neareſt point on the ſector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. diſtances.			
	°	′		°	″	°	′	″	
γ Andromedæ	2	45—	5 31— 6 10	0	31,3	2	44	28,7	North.
β Perſei	1	35—	7 8 7 14	0	6,0	1	34	54,0	N.
δ D°	8	35—	7 32+ 9 13+	1	25,0	8	33	35,0	N.
Capella	7	15+	6 16 4 39—	1	21,3	7	16	21,3	N.
β Aurigæ	6	25+	3 24— 2 22½	1	3,2	6	26	3,2	N.
Caſtor	6	5—	1 10 1 0½	0	9,5	6	4	50,5	South;

		*'s Names.	Nearest point on the sector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. distances.			
1766. October.											
♃	9	Cloudy.	°	'		°	'	°	'	"	
♀	10	α Lyræ	0	5+	3 26½ 0 43½	2	19,0	0	7	19,0	North.
		γ Cygni	1	5—	4 42 6 6—	1	7,7	1	3	52,3	N.
		α D°	6	0—	5 41 5 46—	0	4,7	5	59	55,3	N.
		γ Andromedæ	2	45—	7 12 7 42	0	30,0	2	44	30,0	N.
. . . * . . .		β Perſei	1	35—	8 15½ 8 22	0	6,5	1	34	53,5	N.
		δ D°	8	35—	5 51½ 7 33	1	25,5	8	33	34,5	N.
		Capella	7	15+	6 48½ 5 20½	1	20,0	7	16	20,0	N.
		β Aurigæ	6	25+	3 29½ 2 17½	1	4,0	6	26	4,0	N.
		Caſtor	6	5—	15 25 15 15½	0	9,5	6	4	50,5	N.
♄	11	α Lyræ	0	5+	8 26 5 41+	2	20,7	0	7	20,7	N.
		δ Cygni	6	5+	5 44— 3 37—	1	51,0	6	6	51,0	N.
		γ D°	1	5+	3 37 5 1	1	8,0	1	3	52,0	N.

		✱'s Names.	Neareſt point on the ſector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. diſtances.			
1766. October.			°	′		°	′	°	′	″	
♄	11	α Cygni	6	0+	4 32 4 37½	0	5,5	6	0	5,5	North.
		γ Andromedæ	2	45—	6 14— 6 43	0	29,3	2	44	30,7	N.
		β Perſei	1	35—	7 42— 7 46	0	4,3	1	34	55,7	N.
		δ D°	8	35—	5 38½ 7 17½	1	23,0	8	33	37,0	N.
		Capella	7	15+	8 51 7 23—	1	20,3	7	16	20,3	N.
		β Auriga	6	25+	7 20½ 6 8½	1	4,0	6	26	4,0	N.
		Caſtor	6	5—	4 31 4 20½	0	10,5	6	4	49,5	South.
☉	12	α Lyræ	0	5+	5 29— 2 43—	2	22,0	0	7	22,0	
		δ Cygni	6	5+	4 31½ 2 24½	1	51,0	6	6	51,0	
		γ D°	1	5—	6 26 7 43	1	9,0	1	3	51,0	
		α D°	6	0+	13 8 13 11½	0	3,5	6	0	3,5	
		γ Andromedæ	2	45—	2 37+ 3 18—	0	32,4	2	44	27,6	
· · · * · · ·		β Perſei	1	35—	3 46 3 51	0	5,0	1	34	55,0	

		∗'s Names.	Neareſt point on the ſector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. diſtances.		
1766. October.			°	′		°	′	°	′	″
☉	12	δ Perſei	8	35—	1 43 3 26	1	27,0	8	33	33,0
. . . ∗ . . .		Capella	7	15+	6 28 4 50	1	22,0	7	16	22,0
		β Aurigæ	6	25+	1 33+ 0 21+	1	4,0	6	26	4,0
		Caſtor	6	5—	15 $37\frac{1}{2}$ 15 $29\frac{1}{2}$	0	8,0	6	4	52,0

TURNED THE SECTOR PLANE WEST.

☽	13	γ Andromedæ	2	45—	6 2+ 5 $29\frac{1}{2}$	0	24,8	2	44	35,2
		β Perſei	1	35+	5 43 5 44—	0	0,7	1	35	0,7
		δ D°	8	35—	6 $2\frac{1}{2}$ 4 24	1	22,5	8	33	37,5
		Capella	7	15+	7 $4\frac{1}{2}$ 8 40	1	27,5	7	16	27,5

		*'s Names.	Neareſt point on the ſector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. diſtances.		
1766. October.			°	'		'	"	°	'	"
♂	14	α Lyræ	0	5+	14 12 16 49	2	21,0	0	7	21,0
		α Cygni	6	0	13 16 13 16	0	0,0	6	0	0,0
		γ Andromedæ	2	45—	2 46 2 22	0	24,0	2	44	36,0
		β Perſei	1	35+	2 5— 2 5+	0	0,6	1	35	0,6
· · · * · · ·		δ D°	8	35—	3 27+ 2 0—	1	19,6	8	33	40,4
		Capella	7	15+	1 50+ 3 36	1	29,7	7	16	29,7
		β Aurigæ	6	25+	2 51 4 18½	1	11,5	6	26	11,5
		Caſtor	6	5—	7 17 7 29	0	12,0	6	4	48,0
☿	15	α Lyræ	0	5+	3 33 6 20—	2	22,7	2	7	22 7
		δ Cygni	6	5+	6 18 8 32	1	58,0	6	6	58,0
		γ D°	1	5—	7 4 5 43—	1	5,3	1	3	54,7
		α D°	6	0+	6 36+ 6 37½	0	1,2	6	0	1,2
		Andromedæ	2	45—	6 40½ 6 16	0	24,5	2	44	35,5
		β Perſei	1	35+	5 48— 5 48—	0	0,0	1	35	0,0

		✱'s Names.	Neareſt point on the ſector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. diſtances.		
1766. October.			°	′		′	″	°	′	″
☿	15	δ Perſei	8	35—	8 2 6 24	1	22,0	8	33	38 0
		Capella	7	15+	7 21½ 9 5	1	27,5	7	16	27,5
		β Aurigæ	6	25+	9 25— 10 44½	1	11,8	6	26	11,8
		Caſtor	6	5—	12 45+ 13 6	0	12,7	6	4	47.3
♃	16	α Lyræ	0	5+	4 48+ 7 35	2	22,7	0	7	22,7
		δ Cygni	6	5+	2 20½ 4 32—	1	55,2	6	6	55,2
		γ D°	1	5—	0 25 17 12½	1	4.5	1	3	55,5
		α D°	6	0+	0 8½ 0 9	0	0,5	6	0	0,5
		γ Andromedæ	2	45—	4 31½ 4 3½	0	28,0	2	44	32,0
· · · ✱ · · ·		β Perſei	1	35+	3 25 3 25	0	0,0	1	35	0,0
		δ D°	8	35—	4 4 2 24½	1	23,5	8	33	36,5
		Capella	7	15+	1 20— 3 5	1	29,3	7	16	29,3
		β Aurigæ	6	25+	5 29½ 6 48	1	10 5	6	26	10,5
		Caſtor	6	5—	8 5 8 15½	0	10,5	6	4	49,5

		*'s Names.	Neareſt point on the ſector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. diſtances.		
			°	'		'	"	°	'	"
♀	17	α Lyræ	0	5+	5 18— 8 5	2	23·3	0	7	23,3
		γ Cygni	1	5—	8 1½ 6 41½	1	4,0	1	3	56,0
		α D°	6	0—	6 48½ 6 48	0	0,5	5	59	59,5
		Capella	7	15+	8 43 10 26½	1	27,5	7	16	27,5
		β Aurigæ	6	25+	10 39— 12 6	1	11,3	6	26	11,3
		Caſtor	6	5—	12 14— 12 25+	0	11,6	6	4	48·4
♄	18	α Lyræ	0	5+	6 49 9 38+	2	25,3	0	7	25,3
. . . * . . .		δ Cygni	6	5+	7 45— 10 8	1	59,3	6	6	59,3
		γ D°	1	5—	5 15 4 3	1	4,0	1	3	56,0
		α D°	6	0+	16 47— 16 49	0	2,3	6	0	2,3

THE RESULT OF THESE

Star's zenith diſtance at the point A.

PLANE OF THE

In the tent eaſt.

	D.	α Lyræ. ° ′ ″	D.	δ Cygni. ° ′ ″	D.	γ Cygni. ° ′ ″	D.	α Cygni. ° ′ ″	D.	γ Androm. ° ′ ″
1766.									8	2 44 28,7
Oct.	10	0 7 19,0			10	1 3 52,3			10	2 44 30,0
	11	0 7 20,7	11	6 6 51,0	11	1 3 52,0	11	6 0 5,5	11	2 44 30,7
	12	0 7 22,0	12	6 6 51,0	12	1 3 51,0	12	6 0 3,5	12	2 44 27,6
d h										
Mean 11 11		0 7 20,57		6 6 51,0		1 3 51,7		6 0 4,50		2 44 29,25
Aberration		— 17,11		— 18,40		— 17,33		— 17,75		— 4,08
Nutation - -		+ 6,12		+ 4,14		+ 2,92		+ 2,25		— 7,53
Preceſs. Oct. 11, 1764.		0,0		0,0		0,0		0,0		0,0
Refract. - - -		+ 0,12		+ 6,11		+ 1,06		+ 6,0		+ 2,75
Mean zen. diſt. the 11th Oct.		0 7 9,70		6 6 42,85		1 3 38,42		5 59 55,0		2 44 20,39

PLANE OF THE

	D.	α Lyræ.	D.	δ Cygni.	D.	γ Cygni.	D.	α Cygni.	D.	γ Androm.
October.									13	2 44 35,2
	14	0 7 21,0					14	6 0 0,0	14	44 36,0
	15	0 7 22,7	15	6 6 58,0	15	1 3 54,7	15	6 0 1,2	15	44 35,5
	16	0 7 22,7			16	1 3 55,5	16	6 0 0,5	16	- - - - -
	17	0 7 23,3			17	1 3 56,0	17	5 59 59,5		
		- - - - - -	18	6 6 59,3	18	1 3 56,0	18	6 0 2,3		
Mean - - - -		0 7 22,42		6 6 58,65		1 3 55,55		6 0 0,70		2 44 35,57
Aberration		— 16,74		— 18,31		— 17,40		— 17,93		— 4,79
Nutation - -		+ 6,12		+ 4,14		+ 2,92		+ 2,25		— 7,53
Preceſs. - - -		— 0,03		— 0,14		— 0,18		— 0,19		— 0,19
Refract. - - -		+ 0,12		+ 6,11		+ 1,06		+ 6,0		+ 2,75
Mean zen. diſt. Oct. 11, 1766.		0 7 11,89		6 6 50,45		1 3 41,95		5 59 50,83		2 44 25,81
D° plane eaſt		0 7 9,70		6 6 42,85		1 3 38,42		5 59 55,0		2 44 20,39
Mean zen. diſt. Oct. 11, 1766, at the point A.		0 7 10,79		6 6 46,65		1 3 40,18		5 59 52,92		2 44 23,10

OBSERVATIONS, AS FOLLOWS:

SECTOR EAST.

1766.	D.	β Perſei.		δ Perſei.		Capella.		β Aurigæ.		Caſtor.
		° ′ ″		° ′ ″		° ′ ″		° ′ ″		° ′ ″
Oct.	8	1 34 54,0	8	8 33 35,0	8	7 16 21,3	8	6 26 3,2	8	6 4 50,5
	10	1 34 53.5	10	8 33 34,5	10	7 16 20,0	10	6 26 4,0	10	6 4 50,5
	11	1 34 55,7			11	7 16 20,3	11	6 26 4,0	11	6 4 49,5
	12	1 34 55,0	12	8 33 33,0	12	7 16 22,0	12	6 26 4,0	12	6 4 52,0
		1 34 54.55		8 33 34,17		7 16 20,90		6 26 3,80		6 4 50,62
		— 1,09		+ 2,0		+ 5,48		+ 6,55		— 3,52
		— 8,27		— 8,40		— 7,85		— 7,15		+ 4,70
		0,0		0,0		0,0		0,0		0,0
		+ 1,58		+ 8,55		+ 7,26		+ 6,43		+ 6,08
		1 34 46,77		8 33 36,32		7 16 25,79		6 26 9,63		6 4 57,88

SECTOR WEST.

October.										
	13	1 35 0,7	13	8 33 37,5	13	7 16 27,5				
	14	35 0,6	14	33 40,4	14	16 29,7	14	6 26 11,5	14	6 4 48,0
	15	35 0,0	15	33 38,0	15	16 27,5	15	26 11,8	15	4 47,3
	16	35 0,0	16	33 36,5	16	16 29,3	16	26 10,5	16	4 49,5
					17	16 27,5	17	26 11,3	17	4 48,4
		1 35 0,33		8 33 38,10		7 16 28,30		6 26 11,28		6 4 48,30
		— 1,79		+ 1,20		+ 4,98		+ 6,22		— 3,75
		— 8,27		— 8,40		— 7,85		— 7,15		+ 4,70
		— 0,16		— 0,14		— 0,07		— 0,02		— 0,10
		+ 1,58		+ 8,55		+ 7,26		+ 6,43		+ 6,08
		1 34 51,69		8 33 39,31		7 16 32,62		6 26 16,76		6 4 55,23
		1 34 46,77		8 33 36,32		7 16 25,79		6 26 9,63		6 4 57,88
		1 34 49,23		8 33 37,82		7 16 29,20		6 26 13,20		6 4 56,56

1766.
December.

♃ 11 Set up the ſector at the point N, in the forks of the river Brandiwine, in Pennſylvania, and made the following obſervations.

		✱'s Names.	Neareſt point on the ſector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. diſtances.			
			°	′		′	″	°	′	″	
♄	13	γ Andromedæ	1	15+	5 20½ 4 18—	0	54,8	1	15	54,8	North.
		β Perſei	0	5+	4 46+ 3 18	1	20,3	0	6	20,3	N.
		δ D°	7	5+	4 8 4 6	0	2,0	7	5	2,0	N.
		Capella	5	50—	5 35 8 17	2	18,0	5	47	42,0	N.
☉	14	Cloudy.									
☽	15	γ Andromedæ	1	15+	7 32 6 29	0	55 0	1	15	55,0	N.
		β Perſei	0	5+	8 3 6 25	1	21·7	0	6	21,7	N.
		δ D°	7	5+	6 27 6 24	0	3·0	7	5	3,0	N.
. . . ✱ . . .		Capella	5	50—	4 46½ 7 29½	2	19 0	5	47	41,0	N.
		β Aurigæ	4	55+	9 20½ 6 29½	2	27 0	4	57	27,0	N.
		Caſtor	7	35—	9 14½ 7 35	1	23·5	7	33	36,5	South.

		✱'s Names.	Neareſt point on the ſector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. diſtances.			
			°	′		′	″	°	′	″	
♂	16	γ Andromedæ	1	15+	9 37— 8 33	0	55·7	1	15	55,7	North.
		β Perſei	0	5+	7 45½ 6 16½	1	21,0	0	6	21,0	
		δ D°	7	5+	5 21½ 5 19½	0	2,0	7	5	2,0	
		Capella	5	50—	7 11 9 46—	2	18,7	5	47	41,3	
		β Aurigæ	4	55+	9 39 6 47+	2	27,7	4	57	27,7	
		Caſtor	7	35—	6 34½ 5 5—	1	21,8	7	33	38,2	
☿	17	α Lyræ	1	20+	5 44— 7 42—	1	42,0	1	21	42,0	South.
♃	18	Cloudy.									
♀	19	α Lyræ	1	20+	5 1½ 7 1	1	43,5	1	21	43·5	
. . . ✱ . . .		β Perſei	0	5+	8 42½ 7 14+	1	20,2	0	6	20,2	
		δ	7	5+	7 4 7 0	0	3,7	7	5	3,7	
		Capella	5	50—	5 42½ 8 22½	2	16,0	5	47	44,0	
		β Aurigæ	4	55+	8 6+ 5 15	2	27,3	4	57	27,3	
		Caſtor	7	35—	4 45+ 3 16	1	21,3	7	33	38,7	

		✱'s Names.	Nearest point on the sector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. distances.		
♄	20	Cloudy.	°	'		'	"	°	'	"
☉ ..✱..	21	α Lyræ	1	20+	3 2 5 0—	1	41,7	1	21	41,7

TURNED THE SECTOR PLANE WEST.

☉	21	β Perſei	0	5+	6 4+ 7 42	1	29,7	0	6	29,7
		δ D°	7	5+	7 50— 8 8	0	10,3	7	5	10,3
		Capella	5	45+	4 5— 7 25½	2	56,8	5	47	56,8
			5	50—	4 5— 1 33½	2	7,2	5	47	52,8
		β Aurigæ	4	55+	7 0— 9 51	2	35,3	4	57	35·3
		Caſtor	7	35—	10 0+ 11 38+	1	30	7	33	30,0
☽	22	Cloudy.								
♂	23	D°								
☿	24	α Lyræ	1	20+	6 44— 5 0—	1	36.0	1	21	36,0
		γ Andromedæ	1	15+	4 41— 6 1½	1	4,8	1	16	4,8
		β Perſei	0	5+	4 16½ 6 3—	1	30,2	0	6	30,2
...✱...		δ D°	7	5+	6 16½ 6 27	0	10,5	7	5	10,5

		✱s Names.	Nearest point on the ſector.		Points on the micrometer.	Diff. between the points on the micrometer.		Apparent zen. diſtances.		
1766. Decemb.			°	′		′	″	°	′	″
☿	24	Capella	5	50—	9 19— 6 47	2	7,7	5	47	52,3
. . . *.		β Aurigæ	4	55+	6 2 9 1	2	35,0	4	57	35,0
		Caſtor	7	35—	6 31+ 8 16½	1	29,2	7	33	30,8
♃	25	Cloudy.								
♀	26	D°								
♄ . . * . .	27	α Lyræ	1	20+	5 13+ 3 22	1	35,3	1	21	35,3
		γ Andromedæ	1	15+	8 19 9 31	1	4,0	1	16	4,0
		β Perſei	0	5+	9 17½ 11 3	1	29,5	0	6	29,5
		♪	7	5+	12 16+ 12 26	0	9,7	7	5	9,7
		Capella	5	50—	6 32 4 7	2	9,0	5	47	51,0
	28	γ Andromedæ	1	15+	4 14— 5 26—	1	3,7	1	16	3,7
		β Perſei	0	5+	2 42½ 4 27	1	28,5	0	6	28,5
		β Aurigæ	4	55+	8 14— 11 13+	2	35,6	4	57	35,6

PLANE OF THE

1766. December.		α Lyræ. ° ′ ″		γ Androm. ° ′ ″		β Perſei. ° ′ ″
		....	13	1 15 54,8	13	0 6 20,3
		....	15	1 15 55,0	15	0 6 21,7
		....	16	1 15 55,7	16	0 6 21,0
	17	1 21 42,0		....		....
	19	1 21 43,5		....	19	0 6 20,2
	21	1 21 41,7		....		....
Mean		1 21 42,40		1 15 55,17		0 6 20,80
Aberration		+ 2,20		— 11,76		— 9,20
Deviation		— 5,66		— 7,60		— 8,19
Preceſs.		+ 0,48		— 3,12		— 2,64
Refraction		+ 1,36		+ 1,26		+ 0,10
Mean zen. diſt. Oct. 11, 1766		1 21 40,78		1 15 33,95		0 6 0,87

PLANE OF THE

1766. December.		α Lyræ.		γ Androm.		β Perſei.
		....		....	21	0 6 29,7
	24	1 21 36,0	24	1 16 4,8	24	6 30,2
	27	21 35,3	27	16 4,0	27	6 29,5
		....	28	16 3,7	28	6 28,5
Mean		1 21 35,65		1 16 4,17		0 6 29,48
Aberration		+ 0,15		— 11,63		— 9,56
Deviation		— 5,66		— 7,60		— 8,19
Preceſs.		+ 0,53		— 3,76		— 3,05
Refraction		+ 1,36		+ 1,26		+ 0,10
Mean zen. diſt. Oct. 11, 1766		1 21 32,03		1 15 42,44		0 6 8,78
D° Plane Eaſt		1 21 40,78		1 15 33,95		0 6 0,87
True mean zen. diſt. 11 Oct. 1766, at the point N.		1 21 36,42		1 15 38,19		0 6 4,87
D° at the point A.		0 7 10,79		2 44 23,10		1 34 49,23
Difference		1 28 47,21		1 28 44,91		1 28 44,40
		28 44,91				
		44,40				
		43,98				
		45,09				
		44,34				

Mean = 1 28 44,99 = the true celeſtial arch between the points N and A.

SECTOR EAST.

	δ Perſei.		Capella.		β Aurigæ.		Caſtor.
	° ′ ″		° ′ ″		° ′ ″		° ′ ″
13	7 5 2,0	13	5 47 42,0		. . . .		. . . .
15	5 3,0	15	47 41,0	15	4 57 27,0	15	7 33 36,5
16	5 2,0	16	47 41,3	16	57 27,7	16	7 33 38,2
	. . . .		. . . .		. . . .		. . . .
19	5 3,7	19	47 44,0	19	57 27,3	19	33 38,7
	. . . .		. . . .		. . . .		. . . .
	7 5 2,67		5 47 42,08		4 57 27,33		7 33 37,80
	— 8,61		— 3,20		— 0,60		— 3,78
	— 8,26		— 7,52		— 6,75		+ 4,20
	— 2,26		— 0,95		— 0,29		— 1,25
	+ 7,08		+ 5,80		+ 4,95		+ 7,55
	7 4 50,62		5 47 36,21		4 57 24,64		7 33 44,52

SECTOR WEST.

21	7 5 10,3	21	5 47 52,8	21	4 57 35,3	21	7 33 30,0
24	7 5 10,5	24	5 47 52,3	24	57 35,0	24	33 30,8
27	5 9,7	27	47 51,0		. . . .		. . . .
	. . . .		. . . .	28	57 35,6		. . . .
	7 5 10,17		5 47 52,03		4 57 35,30		7 33 30,40
	— 9,37		— 4,24		— 1,60		— 3,52
	— 8,26		— 7,52		— 6,75		+ 4,20
	— 2,56		— 1,09		— 0,32		— 1,36
	+ 7,08		+ 5,80		+ 4,95		+ 7,55
	7 4 57,06		5 47 44,98		4 57 31,58		7 33 37,27
	7 4 50,62		5 47 36,21		4 57 24,64		7 33 44,52
	7 4 53,84		5 47 40,60		4 57 28,11		7 33 40,90
	8 33 37,82		7 16 29,20		6 26 13,20		6 4 56,56
	1 28 43,98		1 28 48,60 this being a little wide of the reſt is left out.		1 28 45,09		1 28 44,34

The following are Obſervations made at the Points N, and near P, in the Year 1764: But the Length of Time between theſe, and thoſe made at A, being near three Years— probably the Set made at N, in December 1766, may be beſt to be uſed in determining the Length of a Degree of Latitude.

STAR'S ZENITH DISTANCES.

SECTOR IN THE TENT, PLANE EAST.

1764.		γ Andromedæ.	β Perſei.	α Perſei.	♂ Perſei.
		° ′ ″	° ′ ″		° ′ ″
January.	17	1 15 0,0	0 5 40,0	. . . . . .	7 4 29,5
	19	. . . .	0 5 39,6	. . . . . .	7 4 31,0
	20	1 15 2,2	0 5 39,0	9 3 47,0	7 4 30,2
	21	1 15 1,3	0 5 37,5	9 3 49,7	7 4 30,8
	22	1 15 1,2	0 5 38,0	. . . . . .	7 4 31,0
Mean, January	20	1 15 1,2	0 5 38,8	9 3 48,3	7 4 30,5
Aberration declin.		— 10,0	— 9,1	— 11,4	— 10,4
Deviation D°		— 3,3	— 5,7	— 6,2	— 6,7
Preceſs. fr. 1 Jan. 1764.		— 1,0	— 0,8	— 0,7	— 0,7
Refraction		+ 1,4	+ 0,1	+ 10,5	+ 8,3
Obſervatory S° of Tent		+ 0,3	+ 0,3	+ 0,3	+ 0,3
Mean zen. diſt. 1 Jan. 1764		1 14 48,6	0 5 23,6	9 3 40,8	7 4 21,3

SECTOR IN THE OBSERVATORY, PLANE WEST.

		γ Andromedæ.	β Perſei.	α Perſei.	♂ Perſei.
January	26	1 15 4,7	0 5 41,7	. . . . . .	7 4 32,8
♃	27	1 15 5,7	0 5 41,0	9 3 57,0	. . . . . .
	28	1 15 4,5	0 5 44,0	9 3 57,5	7 4 33,0
	29	1 15 5,5	0 5 44,0	9 3 55,3	7 4 32,7
Mean, January	27½	1 15 5,1	0 5 42,7	9 3 56,6	7 4 32,8
Aberration in Declin.		— 9,0	— 8,6	— 11,1	— 10,3
Deviation D°		— 3,3	— 5,7	— 6,2	— 6,7
Preceſs. from 1 Jan. 1764		— 1,3	— 1,1	— 1,0	— 0,9
Refraction		+ 1,4	+ 0,1	+ 10,5	+ 8,3
Mean zen. diſt. 1 Jan. 1764		1 14 52,9	0 5 27,4	9 3 48,8	7 4 23,2
D° Plane Eaſt		1 14 48,6	0 5 23,6	9 3 40,8	7 4 21,3
True zen. diſt. at the point N, 1 Jan. 1764.		1 14 50,8	0 5 25,5	9 3 44,8	7 4 22,2
Preceſs. to Oct. 11, 1766		+ 49,45	40,56	. . . . .	+ 34,61
Reduced to Oct. 11, 1766		1 15 40,25	0 6 6,06	. . . . .	7 4 56,81

*'s Z. DIST. AT THE POINT N.

PLANE WEST.

		Capella		β Aurigæ.		Caſtor		α Lyræ.
		° ′ ″		° ′ ″			Jan.	° ′ ″
1764.	27	5 47 46,8	28	4 57 38,4		....	27	1 21 57,5
	28	5 47 47,0	29	4 57 36,4		....	28	1 21 57,0
January.	29	47 46,7	Feb. 2.	57 38,0		....		
February.	2	47 47,8		....	Feb.	....		1 21 57,3
	3	47 44,7		....	3	7 33 6,6	Ab.	— 9,5
		....	5	4 57 36,8	5	7 33 4,8	Dev.	— 9,4
		....	6	4 57 36,2	6	7 33 5,5	Prec.	+ 0,2
		....		....	8	33 6,8	Refr.	+ 1,5
		....					Feb.	1 21 40,1
		....						
		....					11	1 22 3,8
		....					13	1 22 1,3
Mean	30	5 47 46,6	1	4 57 37,2	5½	7 33 5,9	12	1 22 2,5
Aberration in Declin.		— 7,4		— 5,7		— 0,6		— 13,0
Deviation		— 8,8		— 9,2		+ 9,1		— 9,4
Preceſs. from 1 Jan. 1764		— 0,4		— 0,1		— 0,7		+ 0,3
Refraction		+ 6,7		+ 5,8		+ 8,8		+ 1,5
Mean zen. diſt. 1 Jan. 1764		5 47 36,7		4 57 28,0		7 33 22,5		1 21 41,9
								1 21 40,1
							Mean	1 21 41,0

PLANE EAST.

							Feb.	
February		....	Feb.	....		....	16	1 22 8,2
		....	18	4 57 35,7		....		....
	20	5 47 39,0	20	4 57 34,7	20	7 33 7,3	20	1 22 10,5
	21	5 47 38,7	21	57 35,0	21	7 33 6,3	21	1 22 8,8
	22	47 38,0	22	57 35,2	22	33 5,7	22	1 22 9,8
							26	1 22 10,8
Mean	21	5 47 38,6	21	4 57 35,2	21	7 33 6,4	22	1 22 9,6
Aberration		— 8,0		— 6,9		+ 0,5		— 14,8
Deviation		— 8,8		— 9,2		+ 9,1		— 9,4
Preceſs from 1 Jan.		— 0,7		— 0,2		— 0,95		+ 0,35
Refraction		+ 6,7		+ 5,8		+ 8,8		+ 1,5
Mean zen. diſt. 1 Jan. 1764		5 47 27,8		4 57 24,7		7 33 23.8		1 21 47,3
D° Plane Weſt		5 47 36,7		4 57 28,0		7 33 22,5		1 21 41,0
True zen. diſt. at the point N, 1 Jan. 1764		5 47 32,3		4 57 26,3		7 33 23,1		1 21 44,2
Preceſs. to Oct. 11, 1766		+ 14,56		+ 4,28		+ 18,97		— 7,03
		5 47 46,86		4 57 30,58		7 33 42,07		1 21 37,17

Stars Zenith Diſtances obſerved at a Point 7 chains 91 links North of P.

PLANE EAST.

1764. May.	Capella. ° ′ ″	May	α Lyræ. ° ′ ″
7	6 0 29,8	6	1 9 10,5
8	6 0 28,3	7	9 10,0
9	6 0 29,0	9	10,5
13	6 0 30,0	11	9,0
		12	10,0
		13	7,7
Mean of the 1ſt ſett 10	6 0 29,3	10	1 9 9,6
Aberration	— 1,5		— 12,3
Nutation	— 9,0		— 9,4
Preceſs. fr. 1 Jan. 1764	— 1,9		+ 1,1
Refract.	+ 7,0		+ 1,2
True zen. diſt. 1 Jan. 1764.	6 0 23,9		1 8 50,2

May	δ Cygni. ° ′ ″	May	γ Cygni. ° ′ ″
13	4 50 6,3	13	0 12 59,5
Ab.	+ 14,9		— 15,0
Dev.	+ 8,74		— 8,1
Prec.	— 3,05		+ 4,0
Ref.	+ 5,6		+ 0,2
	4 50 32,5		0 12 40,6
May			
19	4 50 8,0	19	0 13 0,0
20	4 50 9,0	20	12 58,0
23	50 11,3	23	12 57,0
25	50 9,6	25	12 56,0
22	4 50 9,5	22	0 12 57,75
Ab.	+ 13,2		— 13,5
Nut.	+ 8,74		— 8,1
Prec.	— 3,25		+ 4,3
Refr.	+ 5,6		+ 0,2
mean	4 50 33,8		0 12 40,6
Mean of the above.	4 4 32,5		0 12 40,6
True zen. diſt. 1 Jan. 1764, from the mean of all the five obſervations.	4 50 33,15		0 12 40,60

May	α Cygni. ° ′ ″
10	4 43 0,8
11	4 42 58,2
12	43 2,7
13	43 0,0
11½	
mean	4 43 0,4
Ab.	+ 16,5
Dev.	+ 7,6
Prec.	— 4,5
Refr.	+ 5,5
mean	4 43 25,5
May	
19	4 43 1,0
20	43 1,5
23	43 0,5
25	43 2,3
22 mean	4 43 1,3
Ab.	+ 14,9
Nut.	+ 7,6
Prec.	— 4,8
Refr.	+ 5,5
22 mean	4 43 24,5
Mean of the above	25,4
True zen. diſt. 1 Jan. 1764, from the mean of all the obſervations	4 43 25,0

	May	α Lyræ.
	19	1 9 4,3
	23	9 3,8
	25	9 3,7
Mean of the 2d ſett	22	1 9 4,3
Aberration		— 9,5
Nutation		— 9,4
Preceſs.		+ 1,1
Refraction		+ 1,2
Mean		1 8 47,7
Mean above		8 50,2
Mean of all the obſervat. 1 Jan. 1764		1 8 49,0

Star's Zenith Diſtances obſerved at a Point 7 chains 91 links North of P.

PLANE WEST.

1764.		Capella.		α Lyræ.		δ Cygni.		γ Cygni.		α Cygni.
		° ′ ″		° ′ ″		° ′ ″		° ′ ″		° ′ ″
May ⊙	28	6 0 32,7	27	1 08 59,0	27	4 50 14,8	26	0 12 52,6	26	4 43 4,3
June	1	0 32,3	28	8 59,5	28	4 50 51,0	27	12 51,0	27	43 4,2
	5	0 31,7	June		June					
			3	8 58,0	3	50 15,0	28	12 51,0	28	43 4,0
							June		June	
			4	8 59,8	4	50 15,3	3	12 51,0	3	43 7,5
			6	8 58,2	5	50 16,8	4	12 50,3	4	43 5,4
			7	8 57,3	6	50 16,3	5	12 49,8	5	43 6,5
			8	8 57,3	8	50 18,0	6	12 50,0	6	43 7,8
							7	12 49,0	7	43 8,0
							8	12 48,0	8	43 8,3
Mean	1	6 0 32,2	3	1 8 58,4	3	4 50 15,9	3	0 12 50,3	3	4 43 6,2
Aberration		+ 1,4		— 6,3		+ 10,4		— 11,3		+ 12,8
Nutation		— 9,0		— 9,4		+ 8,74		— 8,1		+ 7,6
Preceſs. from 1 Jan. 1764		— 2,2		+ 1,1		— 3,40		+ 4,6		— 5,2
Refraction		+ 7,0		+ 1,2		+ 5,6		+ 0,2		+ 5,5
Mean zen. diſt 1 Jan. 1764 at a point 7 ch. 91 lin.		6 0 29,4		1 8 45,0		4 50 37,24		0 12 35,7		4 43 26,9
D°. Plane Eaſt		0 23,9		1 8 49,0		4 50 33,15		0 12 40,60		4 43 25,0
True zen. diſt. 1 Jan. 1764 at a point 7 cha. 91 lin. North of P		6 0 26,60		1 8 47,00		4 50 35,19		0 12 38,20		4 43 25,95
Preceſs. to 11 Oct. 1766		+ 14,56		— 7,03		+ 23,03		— 30,75		+ 34,59
True zen. diſt. 11 Oct. 1766		6 0 41,16		1 8 39,97		4 50 58,22		0 12 7,45		4 44 0,54
D°. at the point A		7 16 29,20		+ 7 10,79		6 6 46,65		1 3 40,18		5 59 52,92
Difference		1 15 48,04		1 15 50,76		1 15 48,43		1 15 47,63		1 15 52,38

1 15 48,04
15 50,76
48,43 } Arches by different Stars
47,63
52,38

Mean 1 15 49,45 = the celeſtial arch between the point 7 cha. 91 lin. N. of P and the point A

Remarks on re-meaſuring the Lines with two rectangular Levels, or meaſuring Frames.

The levels uſed in this work were, each, 20 feet in length, and 4 feet in height. They were made of pine, an inch thick, and in form of a rectangle; the breadth of the bottom board was $7\frac{1}{2}$ inches, that of the top = 3 inches, of the ends = $4\frac{1}{2}$ inches, and the bottom and top were ſtrengthened with boards firmly fixed to them at right angles. The joints were ſecured with plates of iron, and the ends were plated with braſs. The plumb lines uſed in ſetting them level, were = 3 feet and 2 inches in length, and hung in the middle of the levels, being ſecured in a tube from the wind, in the manner of carpenters levels; wherefore we called theſe by the ſame name.

When the plumb-line biſected a point at the bottom, the ends were perpendicular.

Where the ground was not horizontal, or there were logs, &c. to paſs over, one end of the level was raiſed by a winch and pully.

The level being ſet, a ſhort ſtaff was drove into the ground (very near and oppoſite the plumb-line), in the top of which moved a thin plate of iron, about 12 inches long; at the ends of which were points, which were directed to the interſections of lines, drawn on the board that covered the plumb-line. By bringing the points in a line with one of the ſaid interſections, if the level was by accident moved, it might be diſcovered, and brought again to its place.

A level being thus marked, the end of the other was brought in contact with it, and marked in the ſame manner, before the firſt was moved; the firſt was then taken up, and ſet before the laſt. And ſo the operation was continued. Mr. Dixon attended one plumb-line and ſtaff, and I the other. The meaſure was carried on in a ſtrait line, and in the proper direction, by pointing the levels to the fartheſt part of the viſto that could be ſeen; this was readily and accurately done, on account of their lengths. The levels were frequently compared with the braſs ſtandard, of 5 feet, provided for that purpoſe, and the difference was noted between 8 times the braſs ſtandard, and the length of the two levels taken togerher; as may be ſeen in the 3d and 4th columns of the following table. This difference ſerves for reducing the meaſure taken with the levels, to what it would have been if it had been taken with the braſs ſtandard itſelf; ſee column 6th. For facilitating this compariſon of the levels with the braſs ſtandard, pieces of braſs were fixed into the bottom boards of the levels, on each of which was drawn a faint line. And one tenth of an inch at the end of the braſs ſtandard being divided into ten parts or hundredths of an inch, the difference between eight times the braſs ſtandard, and the two levels joined together, was with the help of a magnifying glaſs of a ſhort focus, determined to great accuracy. Moreover, the braſs ſtandard being liable to alter with the changes of heat and cold, a further correction becomes neceſſary on that account, in order to reduce the meaſures to the temperature of 62° of Fahrenheit's thermometer, which is the term to which the former operations of this kind have been reduced. For this purpoſe, the rate of expanſion of braſs is taken from Mr. Smeaton's experiments, made with a pyrometer of his invention (ſee Philoſ. Tranſ. Vol. XLVIII. Part II.) which is $\frac{232}{10000}$th of an inch upon a length of one foot for a variation of 180° of Fahrenheit's thermometer; whence the expanſion anſwering to four times the length of the braſs ſtandard, or 20 feet, or the length of one level, would be $\frac{464}{1000}$th of an inch for the ſame difference of the thermometer; and $\frac{258}{1000000}$th of an inch for 1° of the ſame thermometer. Therefore, in order to find the correction of column 7th, the conſtant quantity ,00258 was multiplied by the difference of 62, and the degree of the height of the thermometer; and that product, again multiplied by the number of levels meaſured, gave the correction required in inches and decimal parts of an inch; which was additive or ſubtractive, according as the thermometer was higher or lower than 62 degrees.

In the following Proceſs, the

1 and 2 Columns contain the time of the day, M ſignifying morning, A afternoon.

3 - - - - the height of the thermometer at D°.

4 - - - - the quantity, in hundreth parts of an inch, that the two levels, taken together, were more or leſs than eight times the braſs ſtandard, or 40 feet.

5 - - - - the number of levels meaſured between the times, that the levels themſelves were meaſured with the braſs ſtandard.

6 - - - - the corrections, or quantity in inches, to be added to, or ſubtracted from, the number of levels meaſured each day, ariſing from the levels being more or leſs than the braſs ſtandard.

7 - - - - the correction, or quantity ariſing from the thermometer in inches.

Began, at the point N, to re-meaſure the lines with two rectangular levels, 20 feet each in length.

1768 February	1 h	2	3	4	5	6	7
♂ 23	10	M	52	+,12	203	+12,18	— 4,97
	3	A	53	+,12			
		The breadth of Brandiwine the 1ſt time			8,18	+ 0,49	— 0,47
		D° a 2d, where we croſſed			12,20	+ 0,73	

1768 February		1 h	2	3	4	5	6	7	
			D° a third time			9,87	+ 0,39	— 0,20	
	24	9	M	54	+,08				
		1½	A	44	+,12	325	+16,25	—13.41	
		5		39	+,32: :				
	25								
	26	9½	M	40	+,12	247	+14,08	—12,42	
		4½	A	45	+,11				
	27								Rain.
⊙	28								
	29	8½	M	40	+,08	80			Rain in the afternoon.
							+ 3,20	— 0,41	
March	1	9½	M	32	+,23	320	+35,20	—20,64	
		4½	A	42	+,21				
	2	8½	M	32	+,20	200	+15 10	—10,58	
		1	A	51	+,10	170	+ 9,09	— 5 48	
		5½		48	+,115				
	3	8½	M	40	+,185	230	+ 17,82	—10,68	
		2	A	48	+,125	120	+, 8,10	— 4,33	
		5		48	+,145				
	4	8½	M	31	+,19	200	+17,50	—14,19	
		2	A	38	+,16	150	+12,90	—10,83	
		5½		30	+,185				
	5	8	M	27	+,18	220	+16,50	—15.89	
		1	A	41	+,12	120	+ 8 40	— 8.36	
		5½		29	+,165				
⊙	6								
	7	8½	M	28	+,155	250	+15,37	—18,70	Very dry winds with froſt.
		2	A	38	+, 09	90	+ 4 41	— 5,80	
		5¾		36	+,105				
	8	8	M	36	+,103	230	+ 3,8	—10,68	
		1½	A	52	—,037	170	— 1,02	— 5,92	
		5½		45	+, 06				

	1768 March	1 h	2	3	4	5	6	7	
	9	8¼ 2 5½	M A	51 66 52	+,065 —, 11 +,015	270 110	— 3,10 — 2,58	— 2,44 — 0,85	Here ſubtract 4 inches, for going round the corner of a barn.
	10	8¾ 0½	M A	58 61	—,01 —,03	179,2	— 1,79	— 1,15	

This reached the point P.

Hence NP = 3914,45 levels + 199 02 inches — 178,40 inches = 78290 72 feet.

Began at the Point C.

♀	11	11	M	60	—,04	137,15	— 2 74	— 0,71	Rain.
♄	12	9 1 5½	M A	52 58 53	+,07 +,01 +,08	108 156	+ 2,16 + 3,51	— 1,95 — 2,61	
☉	13								
♃	17	Began where we left off, on the 12th inſtant, to meaſure as before.							
		10 2 5	M A	34 40 36	+,09 +,09 +,08	132 144	+ 5,94 + 6,12	— 8,51 — 8,91	
	18	8½ 2 5½	M A	36 42 35	+,085 +,03 +,073	264 156	+ 7,65 + 4,06	—15,66 — 9,45	
	19	9	M Noon	36 31	+,115 +,145	233,35	+14,93	—17,15	

This reached the point D.

Hence CD = 1330,50 levels + 41,63 inches — 64,95 inches = 26608,06 feet.

From D to *g* = 1 chain, and 36 links found before; = 89,76 feet.

☉	20	Sent to Philadelphia, about 40 miles diſtant, for tents, &c.							
♃	24	Began at the point B, to meaſure the line AB.							
		1½	M.	49	—,08	132	— 5,28	— 4,42	
♀	25	. . . .							
♄	26	Noon 6	 A	57 43	—,05 +,065	211,05	+ 1,58	— 6,53	
☉	27								

1768 March		1 h	2	3	4	5	6	7	
☽	28								Snow.
♂	29	8	M	40	+,19	264	+,18,48	—12,60	Compared two thermometers; they agreed within one diviſion.
		2	A	47	+,09	264	+,13, 2	—12,60	
		5½		40	+,11				
☿	30	8½	M	38	+,14	288	+11,95	— 9,28	
		2½	A	61	+,025	192	+ 5,95	— 4,46	
		6¼		45	+,10				
♃	31	8¼	M	45	+,15	204	+12,85	— 4,21	By accident one of the thermom. was broke.
		2	A	62½	+,105	192	+13,15	— 2,97	
		6½		49	+,17				
April									
♀	1	8	M	39	+,195	324	+31,10	—13,79	
		2	A	52	+,19	204	+19.38	— 9,73	
		6		35	+,19				
♄	2	9	M	40	+,18	228	+18,24	—11,17	
		2½	A	46	+,14	168	+12,18	— 8,88	
		6¼		37	+,15				
☉	3								
☽	4	9½	M	38	+,04	312	+12,48	—14,08	Snow in the evening.
		3	A	51	+,12	192	+12,00	— 9,16	
		6		36	+,13				
♂	5								Snow.
☿	6	8½	M	37	+,14	228	+15,39	—10,58	
		3	A	51	+,13				
		6¼		38	+,13	204	+13,26 }	—10,70	
Croſſed Bohemia river obliquely =						33,875	+ 2,21 }		
♃	7								Snow all day, and froſt at night.
♀	8	11	M	37	+,23	396	+38,02	—24,01	
		6¼	A	40	+,155				

1768 April		1 h	2	3	4	5	6	7	
♄	9	9	M	44	+,165	192	+ 7,96	− 5,69	
		2	A	57	00	204	+ 1,02	− 4,21	
		6		51	+,02				
☉	10								
☽	11	9	M	59	−,017	288	−12,81	+ 0,37	
		2½	A	66	−,16	228	−14,82	+ 0,29	
		6½		59	−,10				
♂	12	8	M	47	−,02	348	−13,92	− 3,14	
		3	A	70	−,14	156	− 6,63	+ 1,21	
		6		60	−,03				
☿	13	9	M	64	−,08	240	−11,16	+ 4,95	
		1	A	76	−,105	156	− 6,86	+ 3,22	
		6		64	−,07				
♃	14	8¾	M	53	−,02	396	− 1,98	− 9,19	
		4½	A	53	00				
♀	15	9	M	45	+,18	264	+23,76	−11,57	Rain in the afternoon: this day we paſſed through water, two feet deep, for half a mile.
♄	16	8	M	46	+,20	396	+39,99	−11,23	
		4	A	56	+,205				
☉	17								
☽	18	8¼	M	52	+,11½	264	+ 6,20	− 1,70	
		Noon		67	−,02	192	− 1,44	+ 0,24	
		6½	A	58	−,01				
♂	19	8½	M	53	+,01	444	− 9,50	+ 2,23	
		3	A	74	−,075				
			after 3^h aft.			132			
☿	20	8½	M	57	+,025	312	− 3,12	+ 4,43	
		2½	A	78	−,065	216	− 4,54	+ 4,18	
		6½		61	−,02				

1768 April		1 h	2	3	4	5	6	7	
♃	21	8½ 1½	M A aft. 1½ A	52 75	+,06 +,02	264 132	+ 7,92	+ 1,53	Left off in a ſwamp of water 18 inches deep.
♀ ♄	22 23								Rain day and night.
☉	24								Rain till 11^h A M.
☽ ♂	25 26								Swamps ſo full of water we could not proceed.
☿	27	10 5½	M A	73 72	00 —,035	420	— 3,78	+11,38	
♃	28	8½ 6	M A	54 61½	—,03 —,02	396	— 4,95	— 4,09	
♀	29	. . . .							
♄	30	7 2 6	M A	60 76 73	+,19 +,11 +,11	336 108	+25,20 + 5,94	+ 5,20 + 3 47	
May ☉	1								
☽	2	8½ 6½	M A	54 56	+,05 +,09	384	+13,44	— 6,93	
♂	3	10 3 6¼	M A	61 82 75½	+,115 +,02 +,075	288 108	+ 9,79 + 2,59	+ 7,06 + 4,74	This day we found the diſt. between the 42^d and 43^d mile poſts 3 levels and 6 feet more than uſual between 2 mile poſts.
☿	4	12½	M	79	+,115	228	+13,11	+10,00	Thunderſtorm all the morning: paſſed the main branch of Choptank.

1768 May		1 h	2	3	4	5	6	7	
♃	5	8	M	70½	+,115	264	+ 8,97	+10,90	
		2	A	86	+,02	132	+ 3,63	+ 5,79	
		5½		72	+,09				
♀	6	9½	M	66	+,14	324			
		4	A	63	+,155				
		aft. 4^{h} A				72	+29 11	+ 2,55	
♄	7	8	M	60	+,185	408	+ 25,5	+ 5,26	
		3	A	74	+,065				
☉	8								
☽	9	7½	M	63	+ 03	528	— 8,69	+16,35	
		4½	A	85	—,095				
♂	10	8½	M	61	+,005	372	+ 1,49	+ 2,40	
		7	A	68	+.01				
☿	11	9½	M	70	+,111	528	+22,44	+12,26	Rain in the night and morning.
		6½	A	72	+,06				
♃	12	7	M	54½	+,23	264	+17.16	— 0,68	Rain laſt night.
		2	A	67	+,03	276	+ 3,03	+ 3,92	
		6½		68	+,015				
♀	13	8½	M	68	+,10	300			
		3	A	75	—,025		+ 7,33	+ 9,71	
		aft. 3^{h} A				96			
♄	14	10½	M	66	+,145	264	+18,74	+ 5,45	
		3	A	74½	+,14				
☉	15								
☽	16	7½	M	57	+,19	384			
		3½	A	81	—,02		+22,44	+ 9,53	
		aft. 3½^{h} A				144			
♂	17	8½	M	66	+,12	384			
		3	A	87	—,085				
		aft. 3^{h} A				144	+ 4,75	+19,75	

1768 May		1 h	2	3	4	5	6	7	
☿	18	8½ 1	M A	67 90	+,085 +,14	144	+ 8,13	+ 6,13	Paſſed through Marſhy Hope, water 4 or 5 feet deep.
♃	19	9½ 3	M A	69 86	+,15 +,075	264	+14,78	+10,55	
♀	20	8 3	M A aft. 3^h A	69 93	+,17 +,04	276 168	+23·31	+21,76	Great dews for 4 mornings paſt.
♄	21	9 4	M A	73 86	+,08 —,01	360	+ 6,30	+16,25	Great dew.
☉	22								
☽	23								Rain.
♂	24	7 3	M A aft. 3^h A	50 75	+,35 +,21	480 180	+92,40	+ 0,85	
☿	25	9½ 4	M A	59 56	+,28 +,34	396	+61,38	— 4·59	Rain laſt night and this morning. Paſſed through water part of the day.
♃	26	9½ 5½	M A	58 53	+,38 +,36	528	+97,68	— 8,85	Rain laſt night and part this day, the levels continually wet.
♀	27	8 3¼	M A aft. $3\frac{1}{4}^h$ A	65 79	+,18 +,14	396 264	+52,80	+17,02	Dry weather.
♄	28								
☉	29								
☽	30								

1768 May		1 h	2	3	4	5	6	7	
♂	31	8¼ 3½	M A	79 90	+,01 —,12	376	—10,34	+21,82	Very dry weather for 3 days paſt.
		This reach'd to the North ſide of the river Nanticoke, near to the 7th mile poſt; here we left a mark.							
♃	June 2	Paſſed over the river Nanticoke, and began at the 6th mile poſt from the point A.							
		8 2½	M A	84 74	+,075 +,08	300			
		aft. 2½ʰ A				228	+20,59	+23,15	
♀	3	9 3	M A	76 85	+,155 +,02	396			
		aft. 3ʰ A				264	+29,04	+31,50	
♄	4	6¼ 1	M A	64 82	+,213 00	398,875	+21,15	+11,32	At the point A.
☉	5								
☽	6	Began at the 6th mile poſt, and meaſured Northward through the ſwamp of Nanticoke.							
		7½ 2	M A	67 77	+,12 +,055	230	+10,12	to the river	
		Croſs the river to the mark left the 31ſt of May.				34,67	+ 1,54	6,83	

This finiſhed the line A B.
Hence AB=21696,47 levels +892,34 inches +94,51 inches=434011,64 feet.

The breadth of the rivers was found by meaſuring a baſe, and taking angles with a Hadley's quadrant.

Note. The reckoning was kept by ſtretching a rope in the line to be meaſured (in general)=12 levels, which was often proved: and it was almoſt impoſſible that an error could ariſe; as we always began the rope with the ſame level, and ended it with the other; the rope not being removed till the laſt level was ſet.

The perſon that ſtretched the rope, ſometimes Mr. Dixon, and ſometimes myſelf, kept the account of the number of ropes meaſured: though the mile poſts in the lines AB and DC were ſufficient for that purpoſe, as the lines had been ſo often meaſured before.

In the line NP there were no mile poſts, but two or three intermediate marks, which we found to agree in a general law with the levels.

Suppoſing the levels exactly =20 feet each; then in the line NP a mile per chain meaſure = a mile and 9,44 feet by the levels; and in the line CD a mile per chain meaſure=a mile and 9,86 feet by the levels.

In the line AB, what the levels make more than the chain, between the mile poſts is as follows.

Miles from the point A.	Diff. in feet.	Miles from the point A.	Diff. in feet.
81	14½	43	74
80		42	
78	30½	40	16
76	25	39	8
75	14	38	14-
72	43	37	14
70	32½	36	14½
66	61	35	14
65	15—	34	14½
63	33+	33	14½
61	33½	28	79½
60	16½	27	15
59	15½	25	32
58	16½	24	20½
57	14½	23	14
56	15	22	11½
55	13	21	12½
54	12½	19	22
53	10½	18	14½
52	12½	15	29
51	11	13	14½
50	13	12	5
49	11	10	11½
48	9½	9	8
47	9½	8	7½
45	17½	6	14
44	7	4	23
43	9½	2	13
		0	21½

Here appears to be an error of one chain =66 feet, in the chain meaſure, as obſerved before, p. 318.

We took notice of theſe differences as we meaſured from B to A, always finding the miles greater by the chain meaſure, by the quantity above, which ſhews that the chain was continually extending itſelf by uſe; as we had direct proof of, being obliged to contract it every day, and re-adjuſt it to its proper length by means of the ſtandard chain.

To

To find the Latitude of the Point N.

	δ Perſei	Capella	β Aurigæ	Caſtor	α Lyræ.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
True obſerved zenith diſt. reduced to 1 Jan. 1764.	7 4 22,2	5 47 32,3	4 57 26,3	7 33 23,1	1 21 44,2
Stars decl. by Dr. Bradley.	47 0 40,0	45 43 53,0	44 53 44,2	32 22 56,8	38 34 34,0
Latitude by the different ſtars.	39 56 17,8	56 20,7	56 17,9	56 19,9	56 18,2
	20,7				
	17,9				
	19,9				
	18,2				
Mean = ...	39 56 18,9 = the latitude of the point N.				
Arch between N and A =	1 28 44,9				
	38 27 34 = the latitude of the point A.				
	39 11 56 = the mean latitude.				

Cha. Maſon.
Jere. Dixon.

The Length of a Degree of Latitude in the Province of Maryland and Pennſylvania, deduced from the foregoing Operations; by the Aſtronomer Royal.

THE difference of latitude of the points N and A, or the amplitude of the celeſtial arch, anſwering to the diſtance between the parallels of latitude paſſing through N and A, has been found by the ſector, page 306, to be 1° 28′ 45″,0. The terreſtrial meaſure of the diſtance of the ſaid parallels is next to be found. This is compoſed of the ſum of the lines N P, C D, D*g*, and A R, the laſt mentioned line being the reduction of A B to a meridian line paſſing through A: therefore B R expreſſes a parallel of latitude paſſing through B. Let B*t* be an arch of a great circle drawn perpendicular to the meridian line, A R produced. The triangle B A *t*, on account of the ſmallneſs of its ſides with reſpect to the radius of the earth, and the ſmallneſs of the angle B A *t* =3° 43′ 30″ may be taken for a plane rectilinear triangle, in what follows, without any ſenſible error, as will appear to any one who makes the trial. Therefore it will be, by proportion, as radius is to the coſine of the angle B A *t*=3° 43′ 30″ ſo is A B=434011,6 Engliſh feet, to A *t*=433094,6 Engliſh feet. But this is to be leſſened by the ſmall quantity R *t*, or the diſtance of the parallel circle B R from the great circle

B*t*, which is to a third proportional to the diameter of the earth and the line B R, as the tangent of the latitude of the point B, to the radius. Whence R*t*=15,8 feet which ſubtracted from A *t* juſt found=

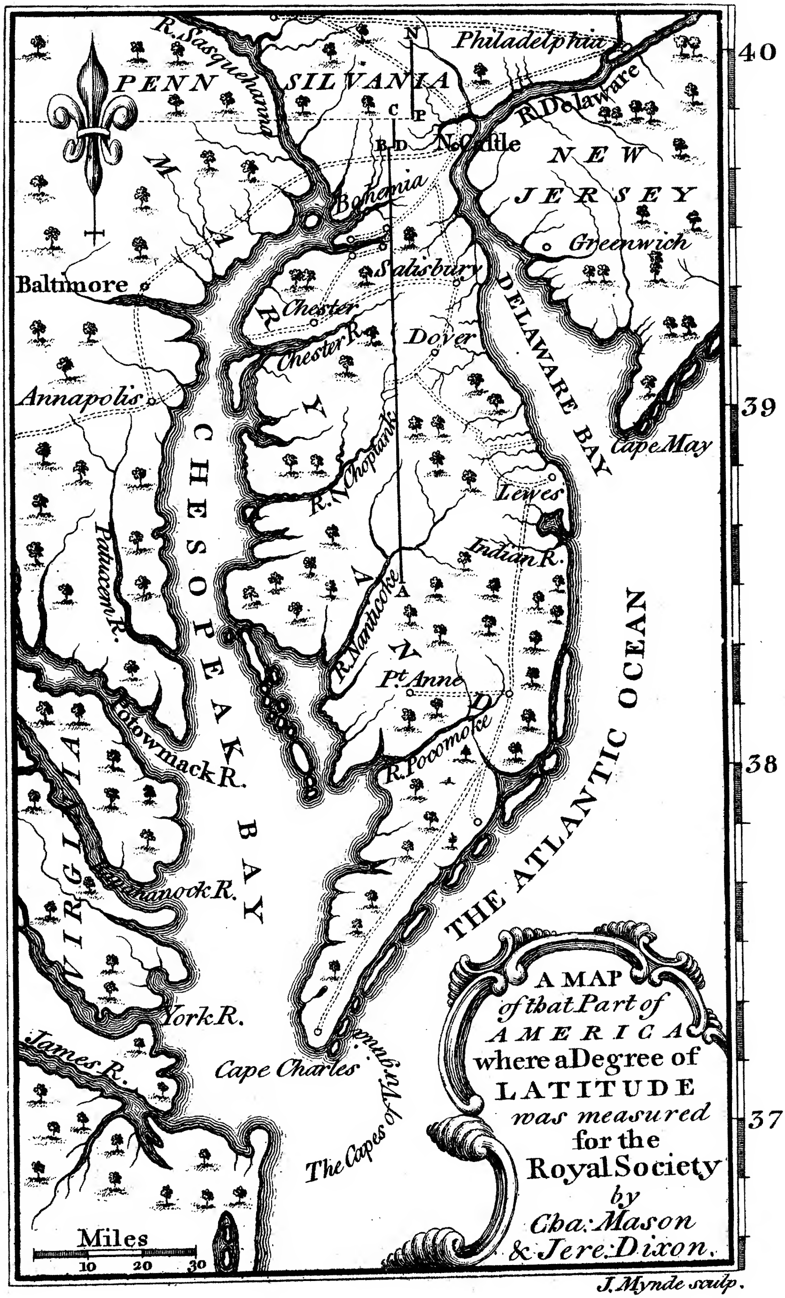
433094,6 leaves	AR=	433078,8 feet
To which add as found before	NP=	78290,7
	CD=	26608 0
	D*g*=	89·7
The ſum is	=	538067 feet

= an arch of meridian intercepted between the parallels of latitude paſſing through the points N and A, anſwering to the celeſtial arch 1° 28′ 45″.

Then ſay, as 1° 28′ 45″; is to 1° : : ſo is 538067 feet, to 363763 Engliſh feet, which is the length of a degree of latitude in the provinces of Pennſylvania and Maryland. The latitude of the Northernmoſt point N, was determined from the zenith diſtances of ſeveral ſtars, ſee page 323=39° 56′ 19″ and the latitude of the Southermoſt point A=38° 27′ 34″. Therefore the mean latitude expreſſed in degrees and minutes is =39° 12′.

To reduce this meaſure of a degree to the meaſure of the Paris toiſe, it muſt be premiſed, that the meaſure of the French foot was found upon a very accurate compariſon, made by Mr. Graham, of the toiſe of the Royal Academy of Sciences at Paris, with the Royal Society's braſs ſtandard, to be to the Engliſh foot, as 114 to 107. See Philoſophical Tranſact, Vol. XLII. p. 185. Therefore ſay as 114 : is to 107 : : ſo is 363763 the meaſure of the degree in Engliſh feet, to 341427 the meaſure of the degree in French feet, which divided by 6, the number of feet in a toiſe, gives the length of the degree=56904$\frac{1}{2}$ Paris toiſes, in the latitude 39° 12′ North.

Such is the length of a degree in this latitude, ſuppoſing the five feet braſs ſtandard made uſe of in this meaſure to have been exactly adjuſted to the length of the Royal Society's braſs ſtandard. It was really adjuſted by Mr. Bird, by his accurate braſs ſcale of equal parts, which he makes ſuch excellent uſe of in dividing aſtronomical inſtruments, and which is juſt $\frac{1}{1000}$th part of an inch ſhorter than the Royal Society's braſs ſtandard upon a length of three feet. If one would take notice of ſo ſmall a difference, the length of a degree juſt found muſt be leſſened by $\frac{1}{36000}$th part, or by ten feet, in order to reduce it to the meaſure of the Royal Society's ſtandard. Since I am treating of ſuch niceties, may it be allowed me to add, that the five feet braſs ſtandard having been again compared with Mr. Bird's ſcale, ſince its return from North America, appeared both to myſelf and Mr. Bird to be juſt $\frac{1}{1000}$th part of an inch ſhorter than the ſcale, upon that ſide on which the hundredths of an inch are placed at one end, and $\frac{2}{1000}$ths of an inch ſhorter than the ſcale upon the oppoſite ſide? which diminution of its length is undoubtedly owing to the ſmall wearing or battering which it has met with in the frequent uſe that was made of it. But the divided ſide of the rod having been that which was made uſe of in meaſuring the levels, is what is to be regarded in the preſent caſe. If one would allow for the wearing of the rod, one may ſuppoſe it to have ſuffered a gradual diminution; and then one muſt take a mean between its firſt length, which was the ſame with Mr. Bird's ſcale, and its preſent length, which is $\frac{1}{1000}$th of an inch ſhorter; as one may ſuppoſe it a medium to have been $\frac{1}{2000}$th part of an inch ſhorter than



A MAP of that Part of AMERICA where a Degree of LATITUDE was measured for the Royal Society by Cha: Mason & Jere: Dixon.

J. Mynde sculp.

than Mr. Bird's ſcale; on which account the length of the degree ſhould be further diminiſhed by $\frac{1}{12000}$th part, or 3 feet, which added to 10 feet, the correction required on account of the difference of Mr. Bird's ſcale and the Royal Society's ſtandard, gives 13 feet to be ſubtracted from the length of the degree calculated above. The whole correction will perhaps be thought ſcarce deſerving of notice, eſpecially as an error of only 1″ in the celeſtial meaſure would produce an error of no leſs than 67 feet in the length of the degree. Moreover it is probable that the length of a degree has been already taken 10 or 20 feet too ſhort, by placing the point C too far to the Southward; which would about balance the ſmall correction in queſtion. Therefore, all things being conſidered, the length of the degree may be ſtated as given above, viz. =363763 Engliſh feet or 56904½ Paris toiſes. It muſt, however, be obſerved, that the accuracy of this reduction into Paris toiſes depends upon a ſuppoſition that the length of the French toiſe, which is of iron, was laid off by the gentlemen of the Royal Academy of Sciences, upon the braſs rod ſent over to them for that purpoſe by Mr. Graham (which was afterwards returned to him); in a room where the heat of the air anſwered to 62 of Fahrenheit's thermometer, or 15 of Reaumur's, or nearly ſo, which is probable enough, but is a point that does not appear to have been aſcertained. For, on account of the difference of expanſion of braſs and iron; 2 rods made of thoſe metals, however accurately they may be made of equal lengths at firſt, will only agree together afterwards in the ſame temperature of the air in which they were originally adjuſted together. It is fortunate that the uncertainty in the preſent caſe is but ſmall, ſince 20° difference of Fahrenheit's thermometer or 10° of Reaumur's produces, according to Mr. Smeaton's experiments, a difference of the expanſions of braſs and iron of only $\frac{1}{13500}$th part, which would cauſe an error of only 27 Engliſh feet or about 4 Paris toiſes in the length of the degree.

It is however to be wiſhed, that the proportion of lengths of the French and Engliſh meaſures might be again aſcertained by another careful experiment, in which the temperature of the air, as ſhewn by the thermometer, might be noted at the time.

[See the Map of the Country, where the foregoing Obſervations were made, TAB. XIV]

POSTSCRIPT, BY THE ASTRONOMER ROYAL.

HAVING, ſome time ago, acquainted M. De la Lande, of the Royal Academy of Sciences at Paris, by letter, of this meaſure of a degree of latitude in North America, and at the ſame time expreſſed my doubts about the certainty of reducing it to French meaſure, from the proportion of the Engliſh to the French foot found by Mr. Graham; principally becauſe no notice had been taken of the height of the thermometer at Paris, when the length of the French iron toiſe was laid off upon the braſs rod ſent thither by Mr. Graham, whence the proportion of the two meaſures was afterwards determined by him; and having alſo mentioned my opinion of the expediency of making another experiment of the proportion of the two meaſures, in which every neceſſary circumſtance ſhould be noted; and that I might probably requeſt the favour of M. De la Lande to take the trouble to cauſe a French toiſe to be made for me, and to ſee it exactly adjuſted to their ſtandard, and then ſent to me; he has been pleaſed to lend me two toiſes, which he ſays are exactly adjuſted to the ſtandard of the toiſe uſed by Meſſ. De la Condamine and Bouguer in the meaſure of the degrees of latitude at Peru, in order to their being compared with the Engliſh meaſure. This compariſon has been made by Mr. Bird, with his uſual accuracy, while I was preſent, and alſo examined the ſame, ſince my account of the length of the degree of latitude aforegoing was printed; and the reſult is, that the longeſt of the two toiſes (for there is a ſmall difference between them), and which has ſince been marked with the letter A, is equal to 76,738 inches by Mr. Bird's braſs ſcale of equal parts, and the ſhorteſt toiſe, which is marked B, is = 76,735 inches by the ſame ſcale; the height of Fahrenheit's thermometer in the room being 61 degrees. The mean of the lengths of the two toiſes is therefore = 76,7365 inches by Mr. Bird's ſcale. But Mr. Bird's ſcale is $\frac{1}{1000}$th of an inch upon 3 feet ſhorter than the Royal Society's braſs ſtandard, and conſequently $\frac{21}{10000}$th too ſhort for the ſame upon 76,7365 inches; therefore $\frac{21}{10000}$th of an inch muſt be ſubtracted from 76,7365; which leaves 76,7344 for the length of the Paris toiſe in meaſures of the Royal Society's braſs ſtandard, in the temperature of 61° of Fahrenheit's thermometer. In the temperature of 62° it will be a little ſhorter; or it may be taken = 76,734 inches in meaſures of the Royal Society's braſs ſtandard. This is $\frac{24}{1000}$th or about $\frac{1}{42}$d of an inch longer than was determined by Mr. Graham's experiment. Hence it appears, that I was miſtaken in ſuppoſing, in p. 325, that the uncertainty about the true proportion of the Engliſh and French meaſures was but ſmall, ſince the error in the former determination now appears to have been $\frac{1}{3197}$th of the whole, or equivalent to what might have been produced by a difference of 84° of Fahrenheit's thermometer. Whence it aroſe I cannot pretend to ſay, neither is it very material to enquire; but the fact is plain, and fully juſtifies the propriety of repeating the experiment.

I ſhall now ſtate the length of the degree, meaſured by Meſſieurs Maſon and Dixon, firſt in Engliſh feet, according to the Royal Society's ſtandard, and then reduced to the French meaſure by the proportion juſt eſtabliſhed.

From 363763 Engliſh feet, the length of the degree found by the 5 feet braſs ſtandard, ſee p. 324, I ſubtract 10 feet for the difference between Mr. Bird's ſcale and the Royal Society's ſtandard, and 3 feet for the wearing of the braſs rod; and there remain 363750 feet, according to the Royal Society's ſtandard, for the length of the degree. But to this it ſeems proper to add 21 feet, in order to correct the poſition of the point C, determined by the ſector, which cannot be ſo certain as that inferred from the azimuth of the line PS. See p. 279. Therefore the true length of the degree, according to the Royal Society's braſs ſtandard, in the temperature of 62° of Fahrenheit's thermometer, is 363771 feet, or 68,8960 Engliſh ſtatute miles. To reduce this to the meaſure of the Paris toiſe, by the proportion above eſtabliſhed, ſay as 76,734 is to 72, ſo is 363771 to 341328 French feet, or 56888 Paris toiſes, of the ſtandard of that uſed in the meaſure of the degrees of the meridian at Peru.

The method made uſe of by Mr. Bird, in finding the length of the toiſes by his ſcale, was as follows, which may ſerve as a direction for the like purpoſe on any future occaſion. Two braſs pins were drove into a ſtrong deal board 4 inches thick, and longer than the toiſe; and two braſs cheeks were made very ſquare, and the ends brought upon the pins. The toiſe was then put in between the cheeks, one of which was made to ſlide ſo as to be eaſily brought into contact with the end of the toiſe, and the other end at the ſame time touching the other cheek, the moveable cheek was ſcrewed faſt; and thus the toiſe was exactly contained between the cheeks without any ſhake, and it is evident that the interval between the cheeks was exactly equal to the length of the toiſe. In order to meaſure this interval, the toiſe being taken away, very fine lines were drawn with a fine point, at the end of each cheek, upon the braſs pins which were in the ſame plain with the board: then the cheeks were removed, and fine points made at the outer extremity of each line, and this diſtance being taken between the fine points of a beam compaſs, was transferred to the ſcale, and thus the length of the toiſe was found in meaſures of the ſcale, which is divided by a vernier to thouſandths of an inch. The toiſes and braſs ſcale had been left together in the ſame room, and near one another all the night before, and till the very time of making the compariſon of the toiſes with the ſcale, in order to be ſure that they were all affected with the ſame degree of heat.

As it may be agreeable to the reader to ſee the reſult of the principal meaſures of degrees of latitude, that have been taken with later inſtruments and proper accuracy, brought together into one view, the following table is here added.

Length of a degree in Paris toiſes.	Mean latitude.			Names of the obſervers.	Years in which the degrees were meaſured.
57422	66°	20′	N	M. de Maupertuis, &c.	1736 and 1737
57074	49	23	N	M. de Maupertuis, &c. and M. Caſſini	1739 and 1740
57091	47	40	N	P. Lieſganig	1768
57028	45	0	N	M. Caſſini	1739 and 1740
57069	44	44	N	P. Beccaria.	1768
56979	43	0	N	Le Pere Boſcowich and Le Maire	1752
56888	39	12	N	Meſſ. Maſon and Dixon . . .	1764 to 1768
56750	0	0		M. Bouguer and M. de la Condamine	1736 to 1743
57037	33	18	S	Abbé de La Caille	1752

If this degree be compared with the degree meaſured at the equator = 56750 toiſes, in the hypotheſis of the earth's being an oblate ſpheroid, the ratio of the equatorial to the polar diameter will come out as 494 to 493. But, if it be compared with the degree meaſured in Laplandy, in the latitude 66° 20′, = 57419 toiſes (I have ſubtracted 3 toiſes, becauſe the toiſe uſed in Laplandy was $\frac{1}{20}$th or $\frac{1}{25}$th of a line leſs than the toiſe uſed in Peru, ſee M. De la Lande's Aſtronomy, Article 2107), the ratio of the diameters will be as 142 to 141. The great difference of theſe reſults is a freſh proof of what has appeared from the compariſon of the meaſures of the ſeveral degrees taken before, either that the figures of the meridians are not accurately elliptical, or that the inequalities of the Earth's ſurface have a conſiderable effect in deflecting the plumb-line from its true ſituation, or both. I had indeed ſuppoſed that any deflections of the plumb-line were not to be feared with reſpect to this particular meaſure of a degree, at the end of my Introduction to Meſſieurs Maſon and Dixon's account of the ſame, by arguing, perhaps too far, from the level diſpoſition of the country through which the degree paſſes. But the Honourable Mr. Henry Cavendiſh has ſince conſidered this matter more minutely; and having mathematically inveſtigated ſeveral rules for finding the attraction of the inequalities of the Earth, has, upon probable ſuppoſitions of the diſtance and height of the Allegany mountains from the degree meaſured, and the depth and declivity of the Atlantic ocean, computed what alteration might be produced in the length of the degree, from the attraction of the ſaid hills, and the defect of attraction of the Atlantic; and finds the degree may have been diminiſhed by 60 or 100 toiſes from theſe cauſes. He has alſo found, by ſimilar calculations, that the degrees meaſured in Italy, and at the Cape of Good Hope, may be very ſenſibly affected by the attraction of hills, and defect of the attraction of the Mediterranean Sea and Indian Ocean.

The rules, which I uſed in calculating the ratio of the equatorial diameter to the polar axis, from the North American degree, compared with thoſe meaſured in Peru and Laplandy, are thoſe given by Mr. John Robertſon, Librarian to the Royal Society, in his Elements of Navigation, p. 597, as deduced by him from Dr. Letherland's Geometrical Analyſis of the problem, which he has alſo given to the public in the ſame place, together with ſome other problems depending upon it, which were neceſſary to complete the ſubject.